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Number of Quanta Required to Form the Photographic Latent Image as Determined from Mathematical Analysis of the H and D Curve. Part II*

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IN a recent paper[1] on this same subject it was shown that theoretical H and D curves of the correct general shape can be obtained by assuming a wide variation of sensitivities among the grains of a photographic emulsion. In this former paper, only conventionalized H and D curves were constructed and the study was carried out by somewhat crude graphical methods. In the present paper analytical methods have been substituted for the graphical procedure in the former paper and the theoretical curves have been compared with experimental curves for actual emulsions. For this purpose three widely differing types of emulsions were used, and comparisons between theoretical and experimental curves have been made in terms of absorbed quanta per grain. The work here, as well as that in the previous paper, has been carried out primarily for the purpose of ascertaining information on the sensitivity distribution among the grains of an emulsion through analysis of the H and D curve, rather than for the purpose of obtaining an equation to represent the H and D curve.

As simplifying conditions for the theoretical work, the emulsion is assumed to consist of grains of uniform size and the gradual decrease in light intensity on passage through the emulsion is not considered explicitly. With regard to this latter effect it is assumed that the total light absorbed is distributed equally (except for statistical fluctuations) among all the grains present in the emulsion. The emulsion is assumed to be made up of grains of varying sensitivity, that is, a fraction α_1, of the grains is considered to require r_1 quanta per grain to acquire developability, a fraction α_2 of the grains to require r_2 quanta to acquire developability, and, in general, the fraction α_m of grains to require r_m quanta to acquire developability.

The formula relating the number of developable grains to the exposure for the grains of one sensitivity class can be obtained from probability considerations and has already been treated fully by Silberstein and others.[2] Mathematically, the problem consists in determining the fraction of grains receiving at least r quanta, when the average grain receives y† quanta. By the

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[1] J. H. Webb, J. Opt. Soc. Am. **31**, 348 (1941).

[2] L. Silberstein, Phil. Mag. **44**, 257 (1922); **44**, 956 (1922); **45**, 1062 (1923); Proc. Int. Math. Congress, Toronto, 1924, p. 127; Phil. Mag. **5**, 464 (1928); L. Silberstein and A. P. H. Trivelli, Phil. Mag. **9**, 787 (1930); T. H. James and J. J. Coleman, J. Chem. Phys. **2**, 483 (1934); T. H. James, J. Chem. Phys. **2**, 1932 (1934); S. P. Shuvalov, J. Phys. Chem. (USSR) **8**, 387, 514 (1936); W. J. Albersheim, J. Soc. Mot. Pict. Eng. **29**, 417 (1937).

† The question as to whether y represents incident, absorbed, or photographically efficient quanta will be left open at present. It is assumed, however, that y is proportional to the incident exposure, i.e., $y=cE$, where E is

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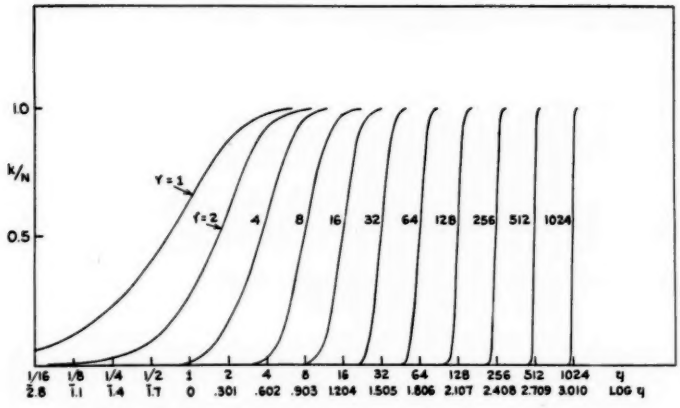

FIG. 1. Theoretical H and D curves computed from Eq. (1) for various constant values of r.

Poisson[3] formula, this can be written

$$\frac{K}{N}=\sum_{n=r}^{\infty}\frac{e^{-y}y^n}{n!}, \tag{1}$$

where K is the number of grains per cm² rendered developable for an exposure y, and N is the total number of grains per cm² of the emulsion. Curves drawn by means of Eq. (1) for various r values are shown in Fig. 1. Curves are shown for grains of one-quantum sensitivity, two-quanta sensitivity, four-quanta sensitivity, etc., up to 1024-quanta sensitivity.

Recently, Silberstein[4] has set down the most general type of quantum formula relating the number of developable grains to the exposure based on the assumption that the emulsion is made up of a great number of different classes of grains, each having a different sensitivity value r. For this case formula (1) is applicable to each class of grains and the total number of developable grains can be obtained by summing the values of K/N for all the different classes. For this purpose it is supposed that the sensitivity of the grains r (minimum number of quanta required to produce developability) extends over the range of integral r values from a minimum value $r_{\min}$ up to some maximum value $r_{\max}$, which may be infinite. The sensitivity distribution among the grains is represented by some function $f(r)$. Thus the fractional number of grains requiring, e.g., one quantum, will be $Nf(1)$, the fractional number requiring two quanta will be $Nf(2)$, etc., and, in general, the fractional number of grains requiring r quanta will be $Nf(r)$. The function $f(r)$ must satisfy the condition

$$\sum_{r=r_{\min}}^{r=r_{\max}} f(r)=1, \tag{2}$$

according to which all fractions of grains must add up to unity. The general expression for the fractional number of grains of all sensitivity classes rendered developable by an exposure y as given by Silberstein is

$$\frac{K}{N}=\sum_{r=r_{\min}}^{r_{\max}} f(r)\sum_{n=r}^{\infty}\frac{e^{-y}y^n}{n!}. \tag{3}$$

If account is taken of the proportionality[5] between number of grains and density, K/N can be replaced by D/D_m, where D is the density ($\log_{10}$ 1/transmission) for an exposure y, and D_m is the maximum density attainable with

the number of quanta per cm² incident on the emulsion during the entire exposure. The maximum value of c is, of course, the area of the grain (a) but its actual value (and thereby the meaning of y) will be left to be determined by a comparison of the theoretical exposure values y with the experimental exposure values E.

[3] T. C. Fry, *Probability and Its Engineering Uses* (D. Van Nostrand, New York, 1928), p. 215.

[4] L. Silberstein, J. Opt. Soc. Am. **31**, 343 (1941).

[5] P. G. Nutting, Phil. Mag. **26**, 423 (1913); G. I. Higson, Phot. J. **60**, 161 (1920); H. Arens, J. Eggert and E. Heisenberg, Zeits. f. wiss. Phot. **28**, 356 (1931); L. Silberstein and A. P. H. Trivelli, J. Opt. Soc. Am. **28**, 441 (1938).

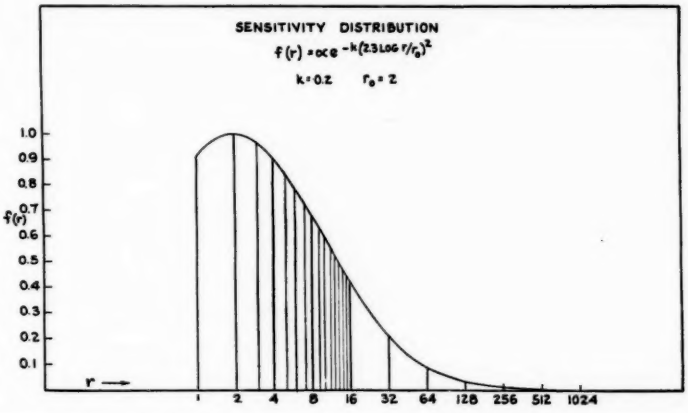

FIG. 2. Illustrative sensitivity distribution curve.

infinite exposure. Thus (3) becomes

$$\frac{D}{D_m} = \sum_{r=r_{\min}}^{r_{\max}} f(r) \sum_{n=r}^{\infty} \frac{e^{-\nu} y^n}{n!}. \qquad (4)$$

Equation (4) expresses the fact that the total density is the summation of a series of H and D curves of the type shown in Fig. 1, where the heights of these various elementary curves have been adjusted in accordance with the sensitivity distribution function $f(r)$.

According to Eq. (4) it may be seen that the problem of determining an equation of the H and D curve resolves itself into that of choosing a form of the function $f(r)$ that will yield theoretical H and D curves in agreement with experiment. A form of the function $f(r)$ that has been found extremely successful in doing this is the function**

$$f(r) = \alpha \exp\left[-k(\ln r/r_0)^2\right], \qquad (5)$$

in which k and r_0 are variable parameters, and α is the normalizing factor necessary to make $f(r)$ satisfy the condition (2). The factor α is given by the expression:

$$\frac{1}{\alpha} = \sum_{r=r_{\min}}^{r_{\max}} \exp\left[-k(\ln r/r_0)^2\right].$$

The parameter r_0 is the sensitivity value for which the sensitivity distribution function $f(r)$ has its maximum value α and k is a parameter governing its over-all latitude. The functional form (5) was first used as a photographic sensitivity distribution function by Lienau[6] (1934). Lienau used the function (5) in its integral form in a purely empirical manner to represent the H and D curve and he referred to his equation as the law of the geometric mean. Later, Houstoun[7] (1936) used the functional form (5) in its integral form to represent the H and D curve. In a still later paper,[8] the author used the functional form (5) directly in a study of the sensitivity distribution function among the grains of several emulsions. In this work the function (5) was used to specify the sensitivity distribution function of each separate size class of grains and the contributions of the grains of separate size class were added together to give the final H and D curve. In all of these former applications, however, the function (5) has been integrated continuously over the whole range of r values between zero and infinity to give the H and D curve. The procedure of using

** This functional form has been used very successfully to represent the sensitivity distribution function for the photoelements of the eye. See for example, E. Hertzsprung, Zeits. f. wiss. Phot. **3**, 468 (1905); R. A. Houstoun, *Vision and Color Vision* (Longmans, Green and Company, London, 1932). It has also been used to represent the size-frequency distribution curves for the grains of a photographic emulsion, see R. P. Loveland and A. P. H. Trivelli, J. Frank. Inst. **204**, 193, 377 (1927). For comments on the mathematical history of the function (5), see A. Fisher, *The Mathematical Theory of Probability* (Macmillan Company, New York, 1923), p. 237.

[6] C. C. Lienau, J. Frank. Inst. **218**, 35 (1934).
[7] R. A. Houstoun, Phil. Mag. **21**, 1113 (1936).
[8] J. H. Webb, J. Opt. Soc. Am. **29**, 314 (1939).

an integral, instead of the summation, is legitimate under certain conditions, as will be shown later in this paper. However, the integration should not start from zero but from some finite integral value of r (1, 2, 3, etc.) corresponding to the most sensitive grains present in the emulsion. In the work to be given here, the quantity r varies from a minimum value $r_{\min}$ (never less than unity) up to a maximum value $r_{\max}$. In all the practical applications made here, $r_{\max}$ has been set equal to infinity. A graph of the function (5) for various integral values of r is shown in Fig. 2. The ordinates for the integral r values have been connected with a smooth curve to show the general trend of the function.

If the function (5) is put into equation (4), the resulting equation is,

$$\frac{D}{D_m}=\alpha\sum_{r=r_{\min}}^{r_{\max}}\exp\left[-k(2.3\log r/r_0)^2\right]\sum_{n=r}^{\infty}\frac{e^{-y}y^n}{n!}, \quad (6)$$

where $(\ln r/r_0)$ has been replaced by its equivalent $(2.3\log_{10} r/r_0)$ and α is to be determined from the equation

$$\frac{1}{\alpha}=\sum_{r=r_{\min}}^{r_{\max}}\exp\left[-k(2.3\log r/r_0)^2\right]. \quad (7)$$

It should be pointed out here that, while (7) is the only formula by which α can be computed accurately, a good approximation formula for α can be obtained by introducing certain modifications in (7) and going over to the integral form. In Fig. 3 the curve (a) represents the function

$$f(r)=\exp\left[-k(2.3\log r/r_0)^2\right],$$

TABLE I. *Values of $1/\alpha$ computed from Eqs. (7) and (8).*

k	r_0	$r_{\min}$	$r_{\max}$	$1/\alpha$ Eq. (7)	$1/\alpha$ Eq. (8)	% Error
0.29	8	4	8	4.78	4.75	0.75
			16	12.29	12.22	.57
			32	23.52	23.39	.51
			64	36.48	36.18	.80
			∞	60.17→61.15	60.64	0→0.85
0.18	2	4	8	4.04	4.04	0
			16	8.47	8.45	0.23
			32	13.77	13.73	.30
			64	19.13	19.07	.30
			∞	29.97→30.86	30.41	0→1.5
0.07	$\frac{1}{16}$	4	8	1.20	1.19	0.84
			16	2.35	2.34	.42
			32	3.71	3.69	.54
			64	5.21	5.19	.38
			∞	12.99→13.27	13.13	0→1.1

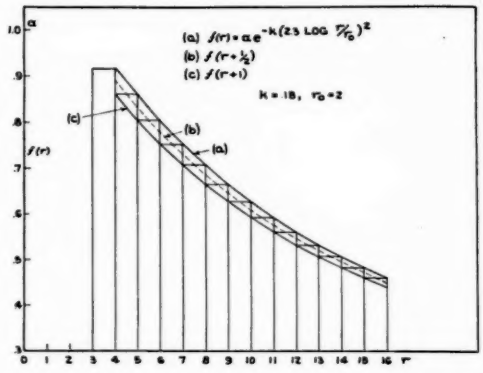

FIG. 3. Illustrative curves to show approximation involved in computing $1/\alpha$ from integral Eq. (8), instead of the summation (7).

with continuous variation of r from $r_{\min}=4$ to $r_{\max}=16$. The stepped rectangular areas, on the other hand, represent the terms of the series (7). It may be readily verified by inspection that a close approximation to the area of the stepped rectangles can be obtained by integrating the function

$$f(r+\tfrac{1}{2})=\exp\left[-k\left(2.3\log\frac{r+\frac{1}{2}}{r_0}\right)^2\right]$$

between the limits $r_{\min}$ and $r_{\max}$ and adding to this quantity, as a constant value, the first term of the series (7). With these changes, the approximation formula for (7) is

$$\frac{1}{\alpha}=\exp\left[-k\left(2.3\log\frac{r_{\min}}{r_0}\right)^2\right]+\int_{r_{\min}}^{r_{\max}}\exp\left[-k\left(2.3\log\frac{r+\frac{1}{2}}{r_0}\right)^2\right]dr,$$

which may be written in the alternative form

$$\frac{1}{\alpha}=\exp\left[-k\left(2.3\log\frac{r_{\min}}{r_0}\right)^2\right]+\int_{r_{\min}+\frac{1}{2}}^{r_{\max}+\frac{1}{2}}\exp\left[-k(2.3\log r/r_0)^2\right]dr. \quad (8)$$

To illustrate how closely Eq. (8) approximates to Eq. (7), values of $1/\alpha$ computed by these two formulas are given in Table I. The values of the constants k, r_0, and $r_{\min}$ used are those found to

apply to the cases of practical emulsions as will appear later in the paper. It may be seen that the percentage error introduced by calculating $1/\alpha$ from Eq. (8) is below one percent for all values of r_{max} up to 64. It is not feasible to compute values of $1/\alpha$ from the summation (7) for values of r_{max} larger than about 32 because of the labor involved. The exact error made by using Eq. (8) for $r_{max}=\infty$ cannot, therefore, be given. It is possible, however, to set upper and lower limits for $1/\alpha$ and to show that the maximum error in $1/\alpha$ computed from (8) is less than a definite numerical value. To do this, we draw the curves $f(r)$ and $f(r+1)$, as shown by the solid lines in Fig. 3. It is obvious that the sum of the areas of the stepped rectangles falls between the areas under these two curves. Likewise, the area under the dashed curve corresponding to the curve $f(r+1/2)$ falls between these two limits. Consequently, if we integrate the functions $f(r)$ and $f(r+1)$ from r_{min} to ∞ and add to each of these as a constant value, the first term of the series (7), upper and lower limits are obtained between which the true value of $1/\alpha$ must lie. By a comparison of the value computed from (8) with these two limiting values, the maximum error is obtained. These extreme values have been calculated for each of the three sets of constants in Table I and are inserted in the column labeled "Eq. (7)" for the $r_{max}=\infty$ value. The maximum error is found to be less than 1.5 percent in every case and the actual error is undoubtedly much less. It should be pointed out that the procedure of computing this maximum error must be modified slightly in case the function $f(r)$ passes through a maximum at some value r above r_{min}. In this case the curves $f(r)$, $f(r+1)$, and $f(r+1/2)$ all cross at a common point near r_0, where the maximum occurs. Under these conditions, the best procedure is simply to add the terms of the series (7) to a point beyond r_0 and then apply the above method, again using the functions $f(r)$ and $f(r+1)$, to determine the maximum error that can arise by substitution of the integration for the summation process from this point on. This process was used in determining the error limits for the first set of constants in Table I.

Equation (6), even with the above approximation equation for $1/\alpha$, is very unwieldy for practical work in fitting theoretical H and D curves to experimental data. It has been found that a simpler working equation can be obtained by making certain further approximations in (6) and (7). In this way, a general equation of the H and D curve, sufficiently accurate for all practical purposes above the toe region, can be obtained in integral form. By means of this equation, values of the constants k, r_0, and r_{min} can be determined fairly readily. The values of these constants so obtained can then be substituted in Eq. (6) and a few points in the toe of the H and D curve calculated. From one or two trials made in this way, values of the constants can be found that give good agreement between theoretical and observed curves throughout. This approximation integral equation will now be derived.

For values of y greater than about ten, it is well known that the Poisson distribution $(e^{-y}y^n/n!)$ occurring in (6) is closely approximated by the normal probability distribution law,[9]

$$1/(2\pi y)^{\frac{1}{2}} \exp\left[-(n-y)^2/2y\right]. \qquad (9)$$

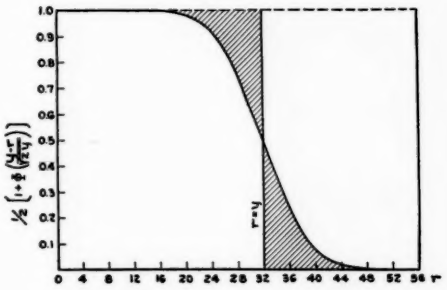

FIG. 4. Graph of function $\frac{1}{2}[1+\Phi(y-r)/(2y)^{\frac{1}{2}}]$ through region around $y=r$.

With the larger values of y, and accordingly n, the variable n can be taken to vary continuously. Therefore (9) can be integrated from $n=r$ to $n=\infty$, and it is readily found that

$$\frac{1}{(2\pi y)^{\frac{1}{2}}}\int_r^\infty \exp\left[-(n-y)^2/2y\right]dn = \frac{1}{2}\left[1+\Phi\frac{y-r}{(2y)^{\frac{1}{2}}}\right], \qquad (10)$$

[9] T. C. Fry, *Probability and Its Engineering Uses* (D. Van Nostrand, New York, 1928), pp. 211 and 239.

where Φ represents the probability integral

$$\Phi(x)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x \exp(-\beta^2)d\beta.$$

Substituting (10) for the summation

$$\sum_{n=r}^{\infty}\frac{e^{-y}y^n}{n!}$$

in (6), the equation

$$\frac{D}{D_m}=\alpha\sum_{r=r_{\min}}^{r_{\max}}\exp\left[-k(2.3\log r/r_0)^2\right]\times\frac{1}{2}\left[1+\Phi\frac{(y-r)}{(2y)^{\frac{1}{2}}}\right], \quad (11)$$

is obtained. The term $\frac{1}{2}[1+\Phi(y-r)/(2y)^{\frac{1}{2}}]$ is shown plotted as a function of r in Fig. 4, for the particular case $y=32$. It may be seen from this graph that this function of r changes from the value unity to zero in the neighborhood of the point $r=y$ and has the value 0.5 at the point $r=y$. The curve is symmetrical on either side of this point, the shaded areas being equal. In the region where y (and accordingly r) is sufficiently large, so that the factor $\exp[-k(2.3\log r/r_0)^2]$ varies only slightly over the range of r values in which the quantity

$$\frac{1}{2}\left[1+\Phi\frac{y-r}{(2y)^{\frac{1}{2}}}\right]$$

changes from unity to zero, the approximation can be made of replacing this quantity by unity and at the same time changing the upper limit of the summation (11) from $r_{\max}$ to y. What is lost by neglecting the terms above y is very closely compensated for by the replacement of the factor

$$\frac{1}{2}\left[1+\Phi\frac{y-r}{(2y)^{\frac{1}{2}}}\right]$$

by unity for all the terms of the summation below y. With these modifications, Eq. (11) becomes

$$\frac{D}{D_m}=\alpha\sum_{r=r_{\min}}^{y}\exp\left[-k(2.3\log r/r_0)^2\right]. \quad (12)$$

This approximation has been found to hold very accurately for values of y greater than 32. In order to determine quantitatively the errors involved in this approximation, values of the summation, for three sets of the constants k, r_0, and $r_{\min}$, have been computed for the two y values 32 and 64 by means of both the accurate Eq. (11) and the approximation Eq. (12). Using values of the constants $k=0.29$, $r_0=8$, and $r_{\min}=4$, and the exposure value $y=32$, the summation (11) yields the value 22.94α while the approximation formula (12) gives 23.42α. Thus the formula (12) is in error by 2.05 percent at this exposure value. For the exposure value $y=64$, the values of the summation computed from formulas (11) and (12) are respectively 36.07α and 36.39α. In this case the error involved by using Eq. (12) is 0.88 percent. This example will serve to show the order of magnitude of the error involved in the approximation formula (12) and the downward trend of this error with higher y values. Further comparison values of the summation computed by Eqs. (11) and (12)

TABLE II. *Values of D/D_m computed from Eqs. (11) and (12).*

y	k	r_0	$r_{\min}$	D/D_m Eq. (11)	D/D_m Eq. (12)	% Error
32	0.29	8	4	22.94α	23.42α	2.05
32	.18	2	4	13.52α	13.77α	1.82
32	.07	$\frac{1}{4}$	4	3.64α	3.70α	1.64
64	.29	8	4	36.07α	36.39α	.88
64	.18	2	4	19.00α	19.13α	.68
64	.07	$\frac{1}{4}$	4	5.17α	5.20α	.57

TABLE IIIa. *Values of D/D_m computed from Eqs. (6), (13), and (15) for values of constants, $k=0.29$, $r_0=8$, $r_{\min}=4$, $r_{\max}=\infty$. Emulsion I.*

y	D/D_m Eq. (6)	D/D_m Eq. (13)	D/D_m Eq. (15)
1	0.0003		
2	.003		
4	.020	0.014	0.0000
8	.078	.078	.064
16	.200	.201	.189
32	.378	.386	.377
64	.593	.597	.592
128		.781	.779
256		.904	.904
512		.967	.967
1024		.991	.991
2048		.998	.998
4096		.999	.999
∞		1.000	1.000

TABLE IIIb. *Values of D/D_m computed from Eqs. (6), (13), and (15) for values of constants, $k=0.18$, $r_0=2$, $r_{min}=4$, $r_{max}=\infty$. Emulsion II.*

y	D/D_m		
	Eq. (6)	Eq. (13)	Eq. (15)
1	0.0007		
2	.006		
4	.038	0.030	0.000
8	.128	.133	.108
16	.274	.278	.259
32	.444	.452	.439
64	.625	.627	.619
128		.776	.772
256		.882	.880
512		.946	.946
1024		.979	.979
2048		.993	.993
4096		.998	.998
∞		1.000	1.000

are shown in Table II. These values were computed for three sets of the constants k, r_0, and r_{min}, which, as will appear subsequently, cover extreme cases of practical emulsions from very low speed to very high speed emulsions. It may be seen that the errors arising from the use of the approximation Eq. (12) vary from 2.05 percent to 1.64 percent for the value $y=32$. For $y=64$, the errors are less, ranging from 0.88 to 0.57 percent. For higher y values, the errors grow steadily less. In the practical applications, the approximation (12) is advocated only for values of $y>32$.

If the summation in Eq. (12) is replaced by the integral from r_{min} to y, and the same refinements made that were used in Eq. (8), the equation

$$\frac{D}{D_m}=\alpha\left\{\exp\left[-k\left(2.3\log\frac{r_{min}}{r_0}\right)^2\right]+\int_{r_{min}+\frac{1}{2}}^{y+\frac{1}{2}}\exp\left[-k(2.3\log r/r_0)^2\right]dr\right\} \quad (13)$$

is obtained, where α is to be computed from the similar equation

$$\frac{1}{\alpha}=\exp\left[-k\left(2.3\log\frac{r_{min}}{r_0}\right)^2\right]+\int_{r_{min}+\frac{1}{2}}^{r_{max}+\frac{1}{2}}\exp\left[-k(2.3\log r/r_0)^2\right]dr. \quad (14)$$

Actually, it has been found unnecessary to make the refinements here that were used in Eq. (8). In the region of y values for which Eqs. (13) and (14) are valid at all, it has been found that the simpler equation

$$\frac{D}{D_m}=\alpha'\int_{r_{min}}^{y}\exp\left[-k(2.3\log r/r_0)^2\right]dr \quad (15)$$

with

$$\frac{1}{\alpha'}=\int_{r_{min}}^{r_{max}}\exp\left[-k(2.3\log r/r_0)^2\right]dr \quad (16)$$

is quite adequate, and approaches closely to the value obtained with (13). This is shown in Tables IIIa, IIIb, and IIIc. Values of D/D_m calculated by means of Eqs. (13) and (15) are given in this table and compared with the values computed from the accurate Eq. (6). It is readily seen from these tables that values of D/D_m computed by Eq. (15), with α' obtained from Eq. (16), are sufficiently accurate for practical work above values of $y=32$. In order to distinguish between the α values calculated by Eqs. (8) and (16), the prime is attached to the one computed with (16).

With a rearrangement of terms, the Eqs. (15) and (16) can be written in terms of the error integral

$$\Phi(x)=\frac{2}{\pi^{\frac{1}{2}}}\int_0^x\exp(-\beta^2)d\beta.$$

TABLE IIIc. *Values of D/D_m computed from Eqs. (6), (13), and (15) for values of constants, $k=0.07$, $r_0=\frac{1}{16}$, $r_{min}=4$, $r_{max}=\infty$. Emulsion III.*

y	D/D_m		
	Eq. (6)	Eq. (13)	Eq. (15)
1	0.0004		
2	.005		
4	.027	0.023	0.000
8	.087	.091	.073
16	.175	.178	.164
32	.278	.281	.271
64	.394	.395	.387
128		.511	.505
256		.623	.619
512		.723	.720
1024		.806	.805
2048		.872	.871
4096		.919	.919
8192		.952	.952
16284		.973	.973
32768		.986	.986
∞		1.000	1.000

The Eq. (16) for $1/\alpha'$ becomes

$$\frac{1}{\alpha'}=\frac{1}{k^{\frac{1}{2}}}\exp\left(2.3\log r_0+\frac{1}{4k}\right)\frac{\pi^{\frac{1}{2}}}{2}\left[\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{r_{\max}}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)-\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{r_{\min}}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)\right]. \tag{17}$$

If this is substituted for α' in (15), the final working equation obtained is

$$\frac{D}{D_m}=\frac{\left[\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{y}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)-\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{r_{\min}}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)\right]}{\left[\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{r_{\max}}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)-\Phi\left(k^{\frac{1}{2}}\,2.3\log\frac{r_{\min}}{r_0}-\frac{1}{2k^{\frac{1}{2}}}\right)\right]}. \tag{18}$$

Equation (18) is incorrect at low exposure values because it neglects entirely the statistical nature of the exposure. According to (18), no grain becomes developable until the average exposure per grain, y, exceeds the threshold value $r_{\min}$ of the most sensitive grains. However, it is well known that, on account of statistical fluctuations, some grains will receive an exposure $r_{\min}$ before the average grain has received an exposure of this amount. Accordingly, some of the spread in the toe of the H and D curve is to be attributed to this statistical fluctuation in exposure from grain to grain. For calculating the shape of the H and D curve in this region, only Eq. (6) can be used. It is to be noted that the sensitivity values, r, of the most sensitive grains of the emulsion, govern the shape of the toe of the H and D curve.

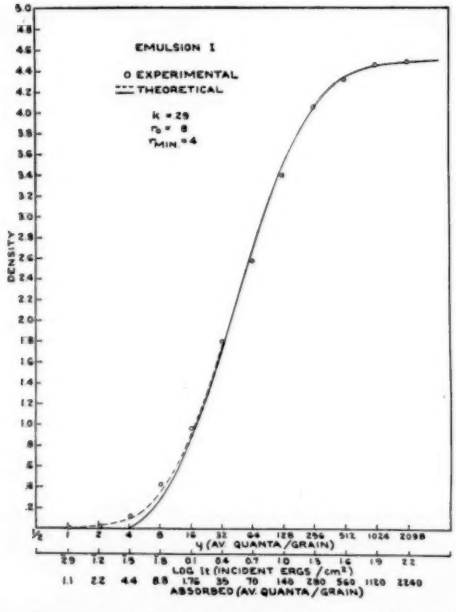

FIG. 5. Comparison of theoretical and experimental H and D curves for emulsion I.

APPLICATION OF THEORETICAL EQUATIONS TO EXPERIMENTAL CURVES

In Figs. 5–7 theoretical curves obtained by means of Eqs. (6) and (18) are shown fitted to observed data. The curves drawn with the solid line were obtained from the approximation Eq. (18), while the partial curves drawn as dashed lines were obtained from the accurate Eq. (6). The same values of the constants k, r_0, and $r_{\min}$ were used in both equations, and their values are specified on the graphs. The numerical values from which these theoretical curves were drawn are given in Table IV. In all three cases, the constant $r_{\max}$ was set at ∞.

The experimental H and D curves, shown by the encircled points in Figs. 5–7, were obtained by time-scale exposures to light of wave-length 4360A. The three emulsions used are tabulated below:

Emulsion I	Low speed	Blue-sensitive
Emulsion II	Medium speed	Blue-sensitive
Emulsion III	High speed	Blue-sensitive

The incident exposure values were measured in absolute units (ergs per cm²) and the logarithms of these exposure values are set down in the middle line of the figures along the abscissa axes. In addition, the exposure values in terms of absorbed quanta per average grain are given in the bottom line of figures along the abscissa axes.

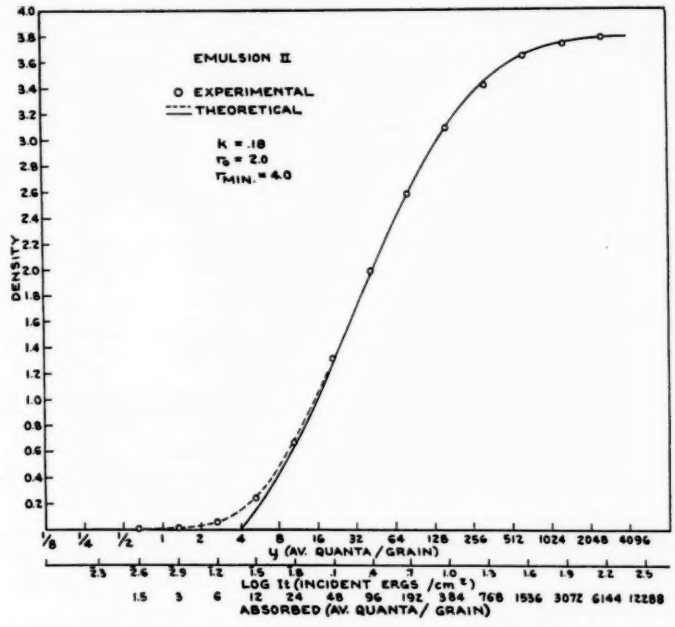

FIG. 6. Comparison of theoretical and experimental H and D curves for emulsion II.

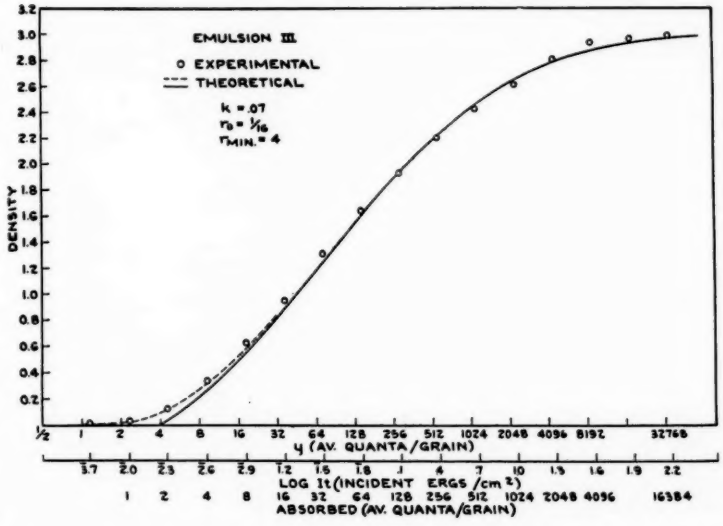

FIG. 7. Comparison of theoretical and experimental H and D curves for emulsion III.

These values are of necessity only approximate and pertain to an average size grain, as shown in Table V. The absorption coefficient was obtained by passing light of wave-length 4360A through the unexposed emulsion and measuring the percentage of diffusely reflected and transmitted

TABLE IV. *Values of densities computed from Eqs. (6) and (18) for emulsions I, II, and III. Values of constants as shown in Figs. 5–7.*

	Densities					
	Emulsion I		Emulsion II		Emulsion III	
y	Eq. (6)	Eq. (18)	Eq. (6)	Eq. (18)	Eq. (6)	Eq. (18)
1	0.00		0.00		0.00	
2	.01		.02		.01	
4	.09	0.00	.14	0.00	.08	0.00
8	.35	.28	.49	.41	.26	.22
16	.90	.85	1.04	.98	.53	.49
32	1.70	1.69	1.69	1.67	.83	.81
64	2.67	2.66	2.37	2.35	1.18	1.16
128		3.50		2.93		1.52
256		4.06		3.34		1.86
512		4.35		3.59		2.16
1024		4.46		3.72		2.41
2048		4.49		3.77		2.61
4096		4.50		3.79		2.76
8192		4.50		3.80		2.86
16384		4.50		3.80		2.92
32768		4.50		3.80		2.96
∞		4.50		3.80		3.00

light. The absorbed light was calculated by subtracting these quantities from unity. To estimate the number of quanta absorbed by an average grain it was necessary to assume that the emulsion is made up of n monograin layers and that $1/n$ of the absorbed light goes to each layer. For this purpose n was assumed to be ten. The number of quanta absorbed by an average grain is obtained by multiplying the light absorbed per unit area in a single layer by the area of an average grain.

From an inspection of the computed and experimental curves of Figs. 5–7, it may be seen that the theoretical curves based on the sensitivity distribution function (5) with the constants shown on the graphs are in close agreement with the experimental points. Furthermore, the theoretical exposure values y (quanta per average grain) are in close agreement with the observed values of absorbed quanta per grain. Here, agreement closer than a factor of two is probably meaningless in view of the manner of determining the absorbed quanta per grain from the experimental data. The agreement obtained as to order of magnitude is, however, considered significant. The exposure values in terms of quanta per grain

TABLE V. *Characteristic constants of emulsions I, II, and III as determined experimentally.*

Emulsion	Average Grain Size (cm²)	Absorption Coefficient	n
I	0.115×10^{-8}	0.55	10
II	0.309×10^{-8}	.52	10
III	0.700×10^{-8}	.65	10

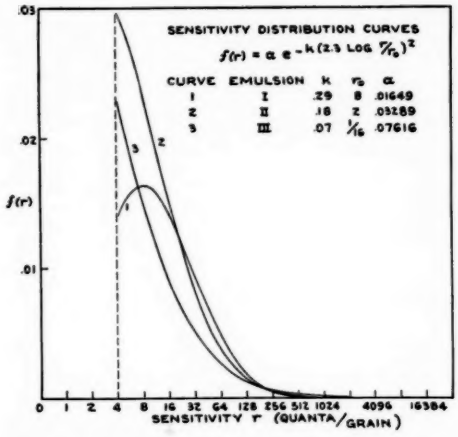
SENSITIVITY DISTRIBUTION CURVES
f(r) = α e^-k(2.3 LOG r/r0)^2
CURVE EMULSION k r0 α
1 I .29 8 .01649
2 II .18 2 .03289
3 III .07 1/16 .07616
.03
.02
.01
f(r)
0 1 2 4 8 16 32 64 128 256 512 1024 4096 16384
SENSITIVITY r (QUANTA/GRAIN)

FIG. 8. Sensitivity distribution curves for grains of emulsions I, II, and III.

as obtained in the theoretical curves indicate that the most sensitive grains in all three of the emulsions tested call for about four quanta. This is indicated by the fact that the value of $r_{\min}$ required in each case to give the correctly shaped toe was four. It might be added here that fitting the theoretical curves to the experimental points in this region affords a very sensitive means for determining ~~the~~ the value of the constant $r_{\min}$. The close agreement between the theoretical exposure values, y, and the measured values of absorbed quanta per grain throughout the H and D curves would seem to

TABLE VI. *Data for the sensitivity distribution function for emulsions I, II, and III calculated from the equation* $f(r)=\alpha \exp[-k(2.3 \log r/r_0)^2]$. *Values of constants* α, k, *and* r_0 *given in Fig. 8.*

	$f(r)=\alpha \exp[-k(2.3 \log r/r_0)^2]$		
r	Emulsion I*	Emulsion II**	Emulsion III***
4	0.143×10^{-1}	0.302×10^{-1}	0.227×10^{-1}
8	0.165	0.233	0.147
16	0.143	0.151	0.899×10^{-2}
32	0.945×10^{-2}	0.827×10^{-2}	0.503
64	0.473	0.380	0.266
128	0.178	0.147	0.132
256	0.511×10^{-3}	0.479×10^{-3}	0.608×10^{-3}
512	0.110	0.131	0.263
1024	0.181×10^{-4}	0.298×10^{-4}	0.106
2048	0.226×10^{-5}	0.588×10^{-5}	0.401×10^{-4}
4096	0.216×10^{-6}	0.961×10^{-6}	0.142
8192			0.477×10^{-5}
16384			0.148
32768			0.419×10^{-6}

* $k=0.29$, $r_0=8$, $\alpha=0.01649$.
** $k=0.18$, $r_0=2$, $\alpha=0.03289$.
*** $k=0.07$, $r_0=\frac{1}{16}$, $\alpha=0.07616$.

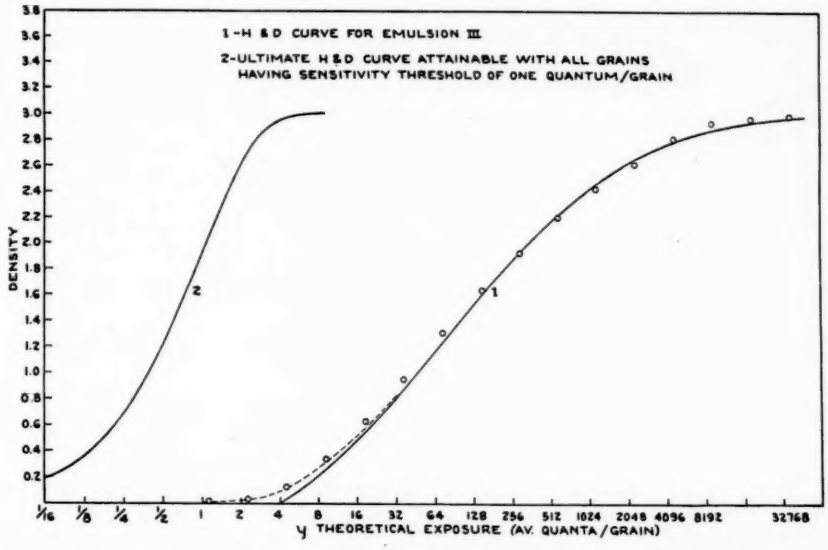
1-H & D CURVE FOR EMULSION III
2-ULTIMATE H & D CURVE ATTAINABLE WITH ALL GRAINS HAVING SENSITIVITY THRESHOLD OF ONE QUANTUM/GRAIN
DENSITY
y THEORETICAL EXPOSURE (AV. QUANTA/GRAIN)
1/16 1/8 1/4 1/2 1 2 4 8 16 32 64 128 256 512 1024 2048 4096 8192 32768

FIG. 9. Comparison of H and D curve observed for emulsion III with theoretical curve of highest ultimate speed.

indicate that the theoretical exposure values, y, are to be identified with the absorbed quanta per grain. It cannot be concluded from this, however, that these absorbed quanta are photographically active quanta. Information on this point can only be obtained from special types of experiments.[10] In the absence of such experimental data, however, it cannot be said that the absorbed quanta are not photographically active.

In Fig. 8, the sensitivity distribution function

$$f(r)=\alpha \exp\left[-k(2.3 \log r/r_0)^2\right],$$

corresponding to the H and D curves of Figs. 5–7 are shown. The numerical values from which these curves were drawn are given in Table VI. Ordinates of these curves represent fractional numbers of grains of each sensitivity class r. In view of the agreement found between the theoretical exposure values, y, and the observed exposure values in absorbed quanta per grain, these sensitivity distribution curves may be considered to be in absolute units of absorbed quanta per grain. The constants k and r_0 specifying the shapes of these curves are given on the graph.

[10] See, e.g., J. H. Webb and C. H. Evans, J. Opt. Soc. Am. **31**, 355 (1941).

From Fig. 7, for the high speed emulsion III, it may be seen that some grains of this emulsion absorb as high as several thousand quanta in acquiring developability. This is shown from the experimental data, irrespective of any theoretical considerations. It is of interest to show in this connection the ultimate attainable speed of an emulsion of the type III if all the grains of the emulsion were equally sensitive and required uniformly the absorption of a single quantum to attain developability. In Fig. 9 are shown two curves for this purpose: Curve (1) is the observed curve of Fig. 7 and curve (2) is a theoretical curve drawn according to Eq. (1), with $r=1$. When it is considered that emulsion III is one of the highest speed emulsions commercially available, it is striking to see what might be done in the way of speeding up emulsions, providing the ultimate theoretical speed could be reached.

ACKNOWLEDGMENT

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The Retinal Mechanism of Color Reception

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(Received July 16, 1941)

This paper is a review of the analysis of color reception with the aid of electrophysiological methods. Microelectrodes have been inserted into the retina to record the discharge of impulses from single or a restricted number of elements in response to illumination with a spectrum of known energy distribution. From the electrodes leads have been taken to amplifier, cathode ray, and loudspeaker. In this manner it has been possible to obtain curves showing the distribution of sensitivity to spectral light of active elements in the eyes of mammals, amphibians, and fishes. Thomas Young's conception, that the retina possesses elements sensitive to different regions of the spectrum, has been proved to be correct. A number of other results illustrate some fundamental properties of the mechanism of color reception.

THE key to the problem of color reception was given by Thomas Young in 1801 when he suggested that there were fibers in the retina carrying impulses elicited by different regions of the spectrum. When he, as well as von Helmholtz 50 years later, for well-known reasons chose a triple response and developed the trichromatic theory, this step was merely an elaboration of his first brilliant generalization. Today we are in a position to put Young's idea to an experimental test. The electrophysiological technique enables us to lead off directly from restricted areas in the retina, and this work has demonstrated the fundamental correctness of Thomas Young's conclusion. At the same time many unexpected facts have turned up and the full significance of them cannot yet be understood. Much remains to be done, but there are already a number of basic facts which deserve to be presented to those outside the domains of electrophysiology who are interested in color vision.

First Attempts with the Electroretinogram

The electroretinogram, a relatively slow polyphasic potential elicited upon illumination of an eye, was discovered in 1865 by the Swedish physiologist Frithiof Holmgren and rediscovered in Scotland somewhat later by Dewar and McKendrick (1873). This discovery raised some hopes that the ancient problem of color vision would yield to the pressure of the new technique. Already in 1901 it was first shown by Himstedt and Nagel that light- and dark-adapted frogs' eyes reproduced the distribution of sensitivity called for by the well-known Purkinje-shift. Other contributions confirmed this conclusion which, however, was not put on a strictly quantitative basis until 1937 by our own work [Granit and Wrede (1937); Granit and Munsterhjelm (1937); Granit (1937)]. It has also been shown that the size of the electroretinogram of dark-adapted eyes was distributed in the spectrum much as the absorption curve for visual purple (see author's review (1938)).

But these facts, after all, did not offer color theory anything that was radically new. Some authors [Kohlrausch (1914), (1918); Brossa and Kohlrausch (1913)] concluded that different wave-lengths gave different types of electroretinogram. And if this had been so there would have been, so to speak, a natural opening for an attack on the physiology of color reception, but these results have been disputed and remain unconfirmed [Graham, Kemp and Riggs (1935); Granit *et al.* (1937)]. They have failed to convince chiefly because many such effects could be imitated by intensity variations, others could be explained on the idea of a difference in the relative contribution of rods and cones to the total response. It is clear that a mixture of photopic (daylight) and scotopic (dim light) spectra can be made to simulate a mixture of, for example, green and yellow.

New hopes were raised by the idea that selective fatigue to one wave-length, by removing its own elements, would leave the electroretinogram with the more or less isolated response to other wave-lengths. Waller (1903) was the first

to try this idea and also the first to be discouraged. In this Journal Chaffee and Hampson (1924) have reported negative results.

Our own work was begun with a close scrutiny of the old results and repetition of experiments of the same general type, first with the simple technique of merely measuring the size of the electroretinogram in an equal energy spectrum, later continued with measurements of the energy necessary for a constant response. Frogs were the only animals used in the beginning. Later other animals, including mammals, were used. For the scotopic and photopic frog's eye the size of the electroretinogram is distributed as in Fig. 1. The two legs of the scotopic curve illustrate physiological variations. The hump in the blue end of the photopic curve proved to vary independently of the rest of the curve. Sometimes scotopic eyes also gave excessively large responses in the blue, when strongly stimulated. The necessity for assuming the presence of a blue-sensitive substance other than visual purple in order to explain such facts was the first objective evidence in favor of Young's theory.

As it seemed possible that failure to obtain positive results in the earlier experiments on selective fatigue might have been due to lack of standard curves based on a sufficient number of data, we repeated those experiments on a large number of eyes. Table I shows the result. The photopic standard, taken immediately after light-adaptation, is compared with measurements of the response to the same wave-lengths *during* illumination of the retina with, respectively, green and red light. In the former case the photopic curve has only become symmetrically narrower, in the latter, however, there is actually selective diminution of the red end. These results were published and discussed in a review written together with W. D. Wright [Wright and Granit (1938)].

TABLE I. *Comparison of size of b-wave, elicited by equal energy spectrum, after light-adaptation (photopic standard) and during adaptation to Ilford spectral red or spectral green.*

WAVE-LENGTH IN μ	PHOTOPIC STANDARD	RED ADAPTATION	GREEN ADAPTATION
0.430	21.5	22	17
0.450	29.3	33	24
0.470	39.4	40	35
0.500	59.3	56	57
0.530	87.0	89	83
0.560	100.0	100	100
0.580	93.3	90	94
0.600	80.0	73	78
0.630	49.3	40	43
0.650	27.0	15	21

On the basis of our early work it was concluded that many results only could be explained on the basis of Young's generalization but that for further advance it was necessary to restrict the number of elements under the electrode in the eye. The electroretinogram, as recorded from the whole retina, is an average of what happens in the active elements. Some isolation of them seemed desirable.

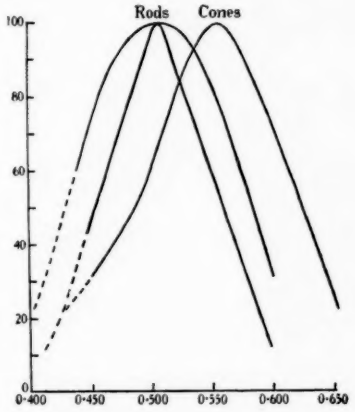

FIG. 1. Size of electroretinogram of photopic (cones) and scotopic (rods) frogs based on a large number of average data. [J. Physiol. **89**, 239 (1937).]

THE MICROELECTRODE TECHNIQUE

Adrian's well-known discovery, that sense organs upon stimulation discharge impulses through their nerve fibers, has led to the recording from several end organs of the frequency of such impulses in response to variations in intensity of the stimulus. Hartline (1938) has recorded from single fibers of the optic nerve of frogs and invertebrates and recently published a review of his results in this journal (1940). We chose a somewhat different approach. A microelectrode was inserted into an eye from which cornea and lens had been removed. If correctly placed on the living retina the microelectrode picks up spikes of impulses from single (or elements synchronized to behave as single) or a

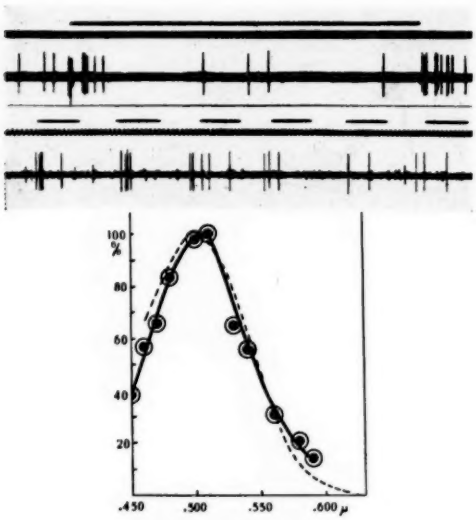

FIG. 2. *Above:* Discharge of single element in rat's eye recorded with microelectrode. In the lower curve the element responds to intermittent light, the upper one shows it to have been spontaneously active. At onset and cessation of illumination the frequency of the discharge is accelerated. Light signal and time in 1/50 sec. also on the film. *Below:* Distribution of sensitivity of the element illustrated in the upper curve. Dotted lines: absorption curve for visual purple. Equal energy spectrum. [Acta Physiol. Scand. **2** (1941).]

restricted number of elements when the eye is illuminated. The spikes (see Fig. 2) can be recorded with any type of fast oscillograph after suitable amplification and listened to in a loudspeaker in the usual manner. The advantage of this technique is that it is easily manageable and can be used *in vivo* with anaesthetized mammals just as easily as with excised eyes of coldblooded animals.

In our work on color reception the general method has been to listen to the discharge in the loudspeaker and observe it on the screen of a cathode ray in order to determine the energy necessary for the threshold or for another constant index such as cessation of "flicker." For details of technique and energy control the reader is referred to a paper by Granit and Svaetichin (1939).

In accordance with results described by Hartline (see his review in this journal, 1940) we also find very many different types of discharge, to illumination as well as to cessation of illumination, but it has not been possible to establish any correlation between type of discharge and color sensitivity. I shall therefore wholly neglect this side of the question, the more so as it has been amply dealt with by Hartline. Only two general observations are of some interest: the first is the very common spontaneous activity, seen in all types of eyes, but most disturbing in mammalian retinae which never are "quiet" [Granit (1941a, c)]; the second is the fact that prolonged activity during illumination is more common in the dark-adapted state whereas light-adapted elements tend to be dominated by brief discharges at onset and cessation of illumination [see also Hartline (1940)]. It is, of course, possible to ascribe the spontaneous activity to the change in intra-ocular pressure after removal of cornea and lens. But seeing that many other sense organs are characterized by spontaneous activity I am inclined to hold the phenomenon to be physiological, perhaps the physiological equivalent of "Eigenlicht." It is indeed probable that normal reception consists in a modulation upon the background of spontaneous activity. In experimental practice it has led to the necessity of measuring just perceptible acceleration of a spontaneous discharge rather than the absolute threshold with mammalian retinae.

A first application of this new technique to the problem of color reception with the frog's eye [Granit and Svaetichin (1939)] soon led to the conclusion that Thomas Young's generalization was correct. But I shall not describe this

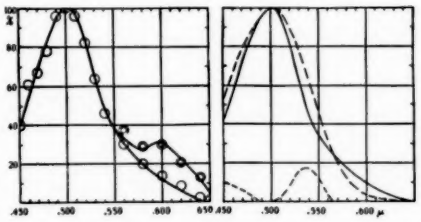

FIG. 3. Curves illustrating the average distribution of sensitivity to spectral light of 10 rats (81 averaged observations from individual experiments, based on in all 152 observations). *To the left:* Equal energy spectrum and curves drawn through the averaged data. *To the right:* The smoothed curve lacking the hump (left) replotted on the basis of an equal quantum intensity spectrum to be compared with Lythgoe's (1937) absorption curve for visual purple (broken lines). Below is plotted the difference between the two upper curves. [Acta Physiol. Scand. **2** (1941).]

work in its chronological order. It is easier to understand if some of the later results are presented first.

Experiments with Albino Rats

This eye is dominated by rods and said to contain only 1 percent cones [Menner (1928); Walls (1934)]. This fact provides an interesting simplification of the whole problem. Figure 2 shows the activity of an isolated element, spontaneously active, and answering with an acceleration of its frequency to onset and cessation of illumination [Granit (1941c)]. In the lower curve it is reacting to intermittent light and the constant index in this experiment was cessation of "flicker." This element, as shown in the same figure, actually gave the distribution of sensitivity required by the absorption curve for visual purple, drawn in dotted lines. A correction to equal quantum intensity of the spectrum would have shifted my curve somewhat to the left.

But very often there was an increased sensitivity in the region of 0.600μ, even in fully dark-adapted rats. When the averages for 10 animals were collected, as in Fig. 3, it was found necessary to split the values in this region into two groups in order to bring out the fact that some elements had a distinct secondary rise around 0.600μ. To the right in Fig. 3 the same curve (without the hump) is replotted in terms of a spectrum of equal quantum intensity in order to compare it with the absorption curve for visual purple (Lythgoe, 1937). The difference between the two

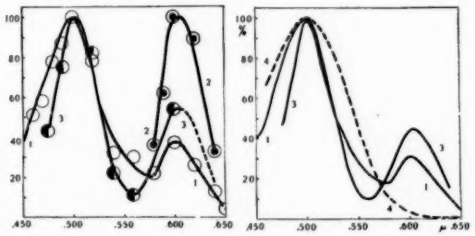

Fig. 4. *To the left:* Visibility curves of an active unit. 1 dark-adaptation, 2 immediately after light-adaptation, 3 somewhat later when the "green" substance is well above the threshold of the monochromator. *To the right:* The same curves similarly labeled corrected for an equal quantum intensity spectrum and compared with 4, the absorption curve for visual purple. [Acta Physiol. Scand. **2** (1941).]

curves, in the region where it must have some significance, is plotted below. This difference-curve rises in the regions where a mixture of hemoglobin and oxyhemoglobin would have absorption maxima. Actually the rat's eye always bleeds a little, when the cornea is removed, and probably therefore the lack of complete correspondence between visual purple absorption and the scotopic spectrum of this eye is due to a thin filter of blood cells.

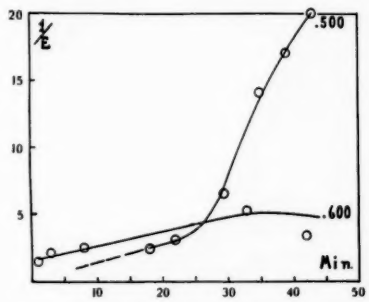

Fig. 5. Shows recovery of sensitivity for the wave-lengths 0.500 and 0.600μ after light-adaptation, in the experiment analyzed in Fig. 4 with regard to the distribution of sensitivity. [Acta Physiol. Scand. **2** (1941).]

Assume now that the eye of the rat, lying with electrode inserted, for 10 minutes is flooded with 2.400 mC. This is very efficient light-adaptation for an eye lacking pigment. Immediately afterwards we follow the recovery of sensitivity (dark-adaptation) for a number of wave-lengths. The result is plotted in Fig. 4. Before light-adaptation curve 1 was obtained with a definite hump in the red. Immediately afterwards only the "red" substance, curve 2, was above the threshold of the monochromator, and thus obtained in the isolated state. It is singularly narrow and precise with a maximum around 0.600μ. But already after a few minutes the "green" substance with its maximum in 0.500μ turns up above the threshold of the apparatus. Somewhat later curve 3 is obtained. To the right in the same figure curves 1 and 3 are compared with the absorption curve for visual purple (curve 4). The "green" curve was unusually narrow already before light-adaptation, despite 36 hours of dark-adaptation of the animal. Afterwards it is still narrower. It is to be noted that all readings are given in percent

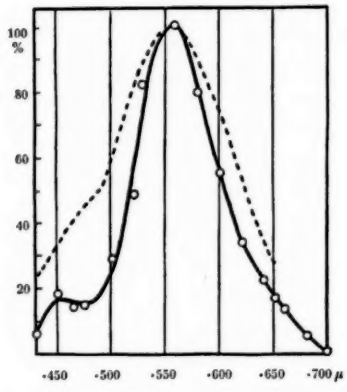

FIG. 6. Inner curve shows distribution of sensitivity of diffuse discharge from frog's eye as recorded with microelectrode of large cross section. Outer curve as photopic curve of Fig. 1 [Upsala Läkaref. Förhandl. N.F. **45** (1939)].

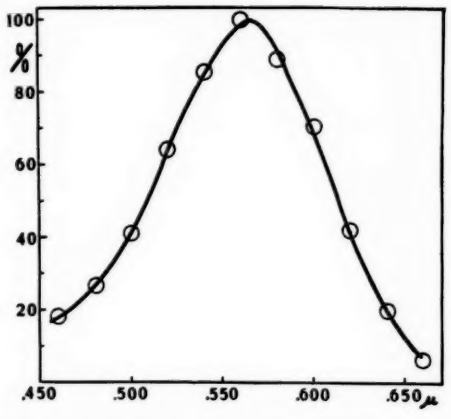

FIG. 7. Distribution of sensitivity of type of element with maximum in 0.560μ in photopic frog's eye. [Acta Physiol. Scand. **3** (1941).]

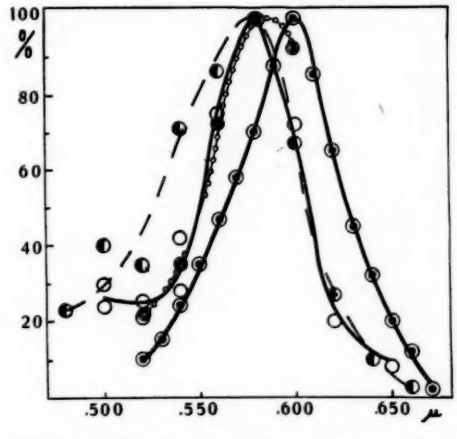

FIG. 8. Distribution of sensitivity of "red" elements from the scotopic frog's eye. [Acta Physiol. Scand. **3** (1941).]

of what at the moment of plotting is the maximum. This is the reason for the shape of the "red" curve 2.

In view of such results it is difficult not to believe that visual purple is the precursor of the "green" substance of the light-adapted eye. Most important from the theoretical point of view is that during the early phase of recovery (dark-adaptation) the "green" curve rises at a slow rate just as the "red" curve. This is shown in Fig. 5. Later on the "green" curve begins to rise at a very much faster rate than the "red" one. This latter phase is what might be called dark-adaptation proper. The characteristic two-step rise of the "green" curve is always noted. It is difficult to believe that the initial slow phase of recovery is anything but a change in a somewhat modified visual purple molecule. In the experiment illustrated in Figs. 4 and 5 the same element to all appearance was active all the time, thus passing through all the changes recorded, and also being sensitive to both red and green. This is what one always finds. The "red" and "green" elements are somehow coupled together at the point where recording takes place. It has not been possible to find isolated spikes merely responding to red.

Actually the albino rat can be taught to distinguish red from green [Walton (1933)] and in my experiments there is obviously a physiological basis for this fact. Assuming the "red" and "green" elements to be the only ones in the rat's eye, what would its color system look like? It would be a yellow- and blue-blind eye in which the "red" and "green" sensitivity bands would be coupled and show much the same adaptive properties in the light-adapted state. Fatiguing the "green" might be expected to leave a predominantly "red" after-image and vice versa. For the rat the sensation of white in darkness would correspond to that of the dark-adapted human eye. It is possible that in the light-adapted the red and green may cancel out by being complementary, and that photopic rats

would see some grayish white. This we do not know.

But how do all these facts fit in with the duplicity theory? It is possible that the "red" element is a cone, coupled to the same nerve fiber as the rods, that is, the element from which the microelectrode picks up the discharge. But to ascribe the "green" substance also to the 1 percent cones would seem to be out of the question. It is far more probable that visual purple under the influence of light is slightly modified, loses its extreme sensitivity and hence acts as a cone substance though still located in the rods.

Actually it is impossible to bleach away the visual purple of an eye [Granit *et al.* (1938, 1939); Lythgoe (1940)]. In the cat's eye, for instance, there is 40 percent visual purple left after a light-adaptation which makes it impossible to elicit an electroretinogram with my monochromator, and there is a definite period of delay before the sensitivity, measured as size of the electroretinogram, enters upon the phase of rapid rise, dark-adaptation proper. The concentration of visual purple begins to rise immediately and long before this phase of dark-adaptation sets in [Granit et al. (1938, 1939)]. There is no proportionality between visual purple concentration and rate of rise of sensitivity, as parallel measurements of the two have fully established. The large amount of "inactive" visual purple, present immediately after light-adaptation, is either merely a store of material waiting for some sort of "intermediate" process determining sensitivity to become completed, or else it has been modified into cone substances [see Lythgoe (1940)]. The experiments with micro-recording from the eyes of albino rats would seem to make the latter assumption unavoidable.

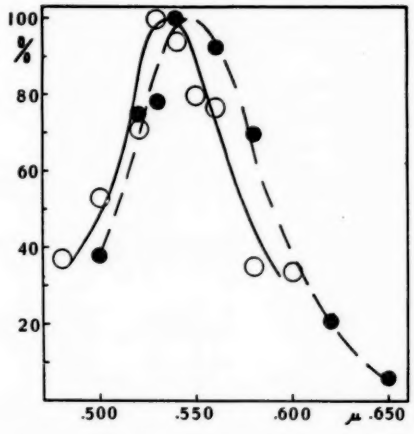

FIG. 9. Distribution of sensitivity of green type of elements in the photopic frog's eye. [Acta Physiol. Scand. 3 (1941).]

A Photopic Spectrum Reminiscent of That of Man

If the cross section of the microelectrode be made very large a great number of elements fall under it, and an average curve can be recorded from the light-adapted frog's eye. A curve of this type is shown in Fig. 6. In dotted lines around it is plotted size of the electroretinogram in an equal energy spectrum. We need not now

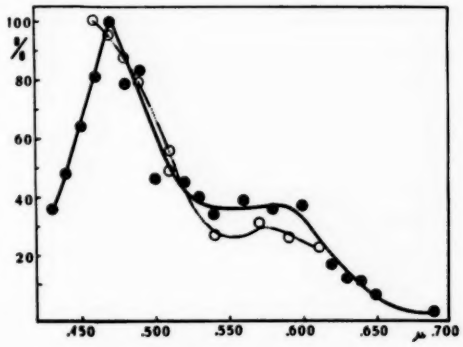

FIG. 10. Blue-sensitive elements in frog's eye. [Acta Physiol. Scand. 3 (1941).]

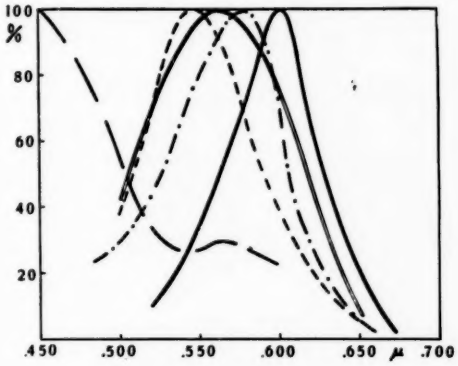

FIG. 11. Elements from light-adapted frog's eye. [Acta Physiol. Scand. 3 (1941).]

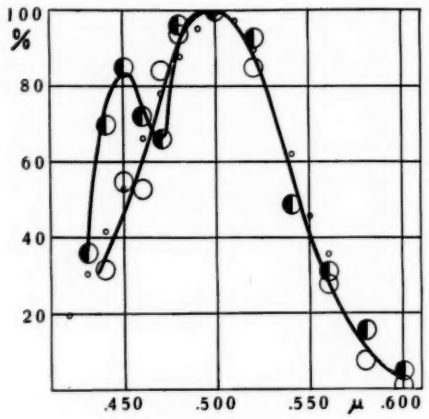

FIG. 12. Averages of records from eye of dark-adapted guinea pig. ○ Averages of values from experiments in which no rise in the blue. ◐ All values from eyes of guinea pigs. The small circles refer to Lythgoe's (1937) absorption curve for visual purple. [Nordisk Medicin (1941).]

enter into the question as to the relation between these two curves [but see Granit and Svaetichin (1939)]. What are the components of this photopic curve which reminds one of the human photopic luminosity curve and possesses a maximum around 0.560μ, where the rat's eye has its minimum? With the frog's eye I possess a large experimental material (well over a hundred retinae), and so should be able to distinguish the unusual from the common. It is to be noted that the microelectrode tends to pick out common types at the expense of unusual ones.

The most common type of element is the one illustrated in Fig. 7. This does not differ much from the average curve for a great number of elements. But there are also other types. Figure 8 shows a number of elements from the "red" end of the spectrum. They are of the narrow type. One of them, a particularly stable one, has its maximum as far out as in 0.600μ. Such types are rare. Somewhat more common are elements with maxima in 0.580μ and between 0.530 to 0.540μ, shown in Fig. 9. The "blue" elements shown in Fig. 10 are rare in the form shown in the figure but quite often readings are found to be influenced by a blue-sensitive substance. Figure 11 gives a summary of these observations.

During dark-adaptation most elements, independently of their initial spectra, sooner or later enter upon a fast rise of sensitivity during which their maxima shift towards 0.500μ. In the course of this change they gradually acquire the shape of absorption curves for visual purple with maxima around 0.500μ. The reason for this is probably the convergence of the paths of rods and cones towards a common final station at or above the point where the microelectrode is located. "Red" elements, which are stable, are sometimes seen but must be regarded as rare. Sometimes, in the fully dark-adapted state, the curves are expanded towards the blue end much further out than could be accounted for by visual purple.

The picture is thus more complicated than in the rat's eye. One of the main questions is clearly whether the curves with maxima around 0.530 to 0.540μ can be regarded as caused by a certain amount of dark-adaptation of elements of the type shown in Fig. 7. I thought so in the beginning [Granit and Svaetichin (1939)] but, having since in the eye of light-adapted guinea-pigs found elements with maxima between 0.530–0.540μ, I think they may well be genuine in the frog's eye too. In the guinea pig I have found no element with maximum further out towards the long wave-lengths than in 0.530 to 0.540μ, and even the latter ones are not very common. In the frog's eye I have not found the narrow curve with maximum in 0.500 which is seen in rats and guinea pigs.

The eye of the guinea pig is of particular interest because it has a very marked "blue" element which influences the average curve shown in Fig. 12 just as the "red" element often

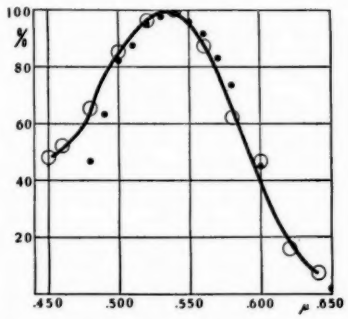

FIG. 13. Averages of observations from eyes of dark-adapted tenches. Small black dots absorption curve for visual violet according to Bayliss, Lythgoe and Tansley (1936). Spectrum of equal quantum intensity. [Acta Physiol. Scand. **2** (1941).]

shows up in the average curve for dark-adapted frogs [Granit (1937)].

In view of the results with the frog's eye, would a trichromatic theory seem possible for this animal? Birukow (1939) has shown that the frog has good color vision but that it may be somewhat green-blind. Clearly some polychromatic theory would be more in accordance with the experimental results but, on the other hand, it would perhaps be possible to average the curves with maxima between 0.580–0.600μ into one set, those around 0.530–0.540μ into another, and, finally, the "blue" ones into a third group of elements and thus to work out a trichromatic theory for the frog's eye. The broad type of curve of Fig. 7 which is so common in the frog's eye could be regarded as a combination of the "red" and the "green" substances, a combination which may be photochemical or physiological. In the former case the substances would be mixed in the sensory cells, in the latter "red" and "green" elements with narrow sensitivity bands would be coupled together physiologically

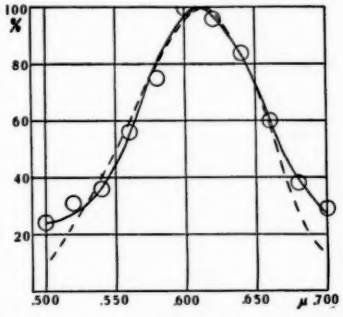

FIG. 14. Averages of observations from eyes of light-adapted tenches. The dotted line shows distribution of sensitivity of isolated red element of the Greek tortoise. Spectrum of equal quantum intensity. [Acta Physiol. Scand. **2** (1941).]

at or above the point where recording takes place.

The sum total of active elements would then give the daylight spectrum of the frog's eye, shown in Fig. 6. It is likely that the relative ordinates which one would be forced to choose in order to be able to combine the average curve from the narrow elementary curves, would be determined by the total number of elements of each type rather than by their relative sensitivity. Assuming the frog to see white and a number of colors, photopic white would probably be determined by the total impulse frequency in the activated elements. Probably the broad curve of Fig. 7 also would represent white, and colors would be determined by the different narrow bands serving, so to speak, as modulators of white. Color discrimination would be great in those parts of the spectrum where the number of slightly different narrow modulators is great.

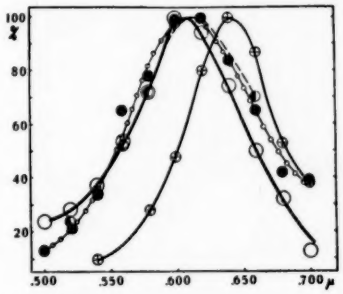

FIG. 15. Individual curves from the retinae of photopic tenches. [Acta Physiol. Scand. **2** (1941).]

The rather special position of the blue is obvious. It is quite possible that the scotopic white of an eye saturated with visual purple, has a bluish tinge.

In many respects there is great likeness between the visual systems of frogs and man. How far the likeness goes, we cannot say, but to those acquainted with the subject of color vision it is clear that the results obtained with the frog's eye *could* form a basis for an analysis of the mechanism of color reception in man. Only in one respect was there a better fit in the results obtained with the rat's eye. This was the coupling of a "green" and "red" element in this eye.

THE RELATION BETWEEN ROD AND CONE SUBSTANCES

Certain fishes possess a type of visual purple which is called visual violet and has the maximum of its spectrum between 0.530–0.540μ, when calculated from a difference spectrum [Koettgen and Abelsdorff (1896); Grundfest (1932); Bayliss, Lythgoe, and Tansley (1936); Wald (1939)]. The photopic and scotopic spectra of one such fish (*Lepomis*) were studied by Grundfest (1932a, b) by a behavioristic method based on

the reaction to a moving pattern. The scotopic spectrum had a maximum around 0.540μ, the photopic one around 0.600μ, both spectra being somewhat distorted by what Grundfest held to be absorbing filter-substances in the eye. Still, the important point in this experiment was that the photopic spectrum was shifted towards the red end in much the same proportion as visual violet was shifted by comparison with visual purple. Today we know that the photopic spectrum is complex and that even in the frog's eye, which has the mammalian type of visual purple, individual sensitivity bands may reach as far out as into 0.600μ, though the average photopic curve has its maximum in 0.560μ. Grundfest's very good argument, that proportionate shifts of the photopic and scotopic curves indicate chemical interrelationship between rod and cone substances therefore needs reinvestigation with the electrophysiological technique. A weakness in his argument is also evinced by the possible rejoinder that fishes as a species may have cones more sensitive to red than frogs and mammals. Since the work of Bayliss, Lythgoe and Tansley (1936) we know that there are also fishes with the visual purple of mammals and frogs. An analysis of the photopic spectrum of fish of this type would therefore be of great interest.

Work with the microelectrode technique has brought clarity into this question. As representative of fish with visual violet we shall use the tench (*Tinca*). The fresh-water eel (*Anguilla*) will serve as representative of fish with visual purple. The average scotopic curve for dark-adapted tenches, compared with the absorption curve for visual violet, is shown in Fig. 13. In Fig. 14 follows the average curve for the light-adapted fish. The extreme red sensitivity of photopic tenches is obvious. A set of similar curves could be obtained from the eyes of the carps which also possess visual violet. The curve drawn in dotted lines in Fig. 14 represents the isolated "red" receptor of the Greek tortoise (*Testudo*) which has a practically pure cone eye, apparently with properties of the kind found in photopic fishes with bleached visual violet. This interesting fact shows that there are *preformed* cone substances of the type found after light-adaptation of fishes. The "red" element of the tortoise is easy to isolate. Only once I have

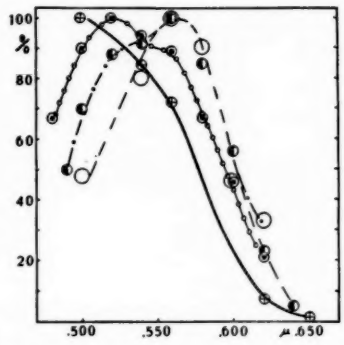

FIG. 16. The individual curves from the eyes of the fresh-water eel. [Acta Physiol. Scand. **2** (1941).]

found a pure "green" element (0.530μ) in this animal, but its average photopic curve shows that the "red" element cannot alone be responsible for its color reception [Granit (1941b, d)].

The average photopic curve of Fig. 14 is a mixture of components of the type shown in Fig. 15. Thus in the photopic tench there is a family of curves just as in the frog's eye and with maxima spread over a large spectral range.

In the retina of the eel it proved impossible to find the extremely red sensitive elements which dominate the photopic retinae of fishes possessing visual violet. Figure 16 illustrates the type of elements which are found in the eye of the fresh-water eel where visual violet is replaced by visual purple. None of them goes beyond 0.560μ.

It is therefore safe to conclude that a shift of the scotopic curve runs parallel with a shift of the photopic curve in the same direction, independently of the species studied. It would be unwise to overemphasize the strict proportionality in these Purkinje-shifts counted from the scotopic maximum. We have seen that the photopic elements may have narrow absorption bands as widely apart as 0.050μ. In some eyes there may be more of one type, in others more of another. Thus in the rat's eye I found only a single narrow "red" photopic band with maximum in 0.600μ. It is therefore best to be content with the statement that the further out towards the long wave-lengths the spectrum of the scotopic substance is located, the greater the probability for the occurrence of photopic bands

in the extreme red. These facts are difficult to explain unless it be assumed that the scotopic and photopic substances are chemically interrelated.

Wald (1935) and Krause and Sidwell (1938) have shown that visual purple consists of a protein carrier and a prosthetic group. Wald's experiments have made it likely that the prosthetic group is a carotinoid. He calls it retinine. Illumination breaks the linkage between protein and carotinoid and sets free a yellow product, by Lythgoe (1937) called "indicator yellow," because its absorption spectrum, also determined by him, shifts with the *p*H of the solution. Retinine and visual yellow are closely related products. Wald (1938, 1939) finds that retinae containing visual purple in the bleached state contain Vitamin A_1 whereas retinae with visual violet contain Vitamin A_2. In the latter case the retinine formed is of a deeper yellow tone, retinine$_2$. The active systems are thus similarly organized for visual purple and visual violet, and differ only by being throughout displaced in the spectrum relative to one another. We have seen now that this displacement also turns up in the equivalent cone systems. All these facts give further support to the conclusion that the rod and cone systems are chemically interrelated.

How could we imagine the cone systems to be formed from the rod systems or, possibly, vice versa? Broda, Goodeve and Lythgoe (1940) have shown for visual purple that each protein molecule carries about 10 chromophoric groups. In view of this, one is entitled to assume that small changes, say, in the linkage between chromophores and carrier suffice to constitute the many types of cone substances described in this paper. It is true that none so far has been definitely isolated but this need not deter us from putting forth this hypothesis. In favor of it there is also the evidence by Hecht and Mandelbaum (1939) that not only rod adaptation but also cone adaptation is influenced by lack of Vitamin A. I should perhaps mention that v. Studnitz (1937, 1940) claims to have isolated cone substances from the eyes of frogs, fishes, and the Greek tortoise. In all cases he finds a maximum in 0.560μ. Actually, however, only frogs possess a maximum in this region whereas fishes and tortoises have their average maxima between 0.600–0.620μ. His methods cannot therefore have been adequate enough to support the conclusions which he has advocated.

Bleaching of visual purple leads to other yellow substances than the indicator yellow or retinine. The latter is rather insensitive to light. But, for instance, Lythgoe (1937) and Lythgoe and Quilliam (1938) have described a photoproduct which they call "transient orange" and which may have something to do with the perception of blue and the special position it occupies by virtue of its rod-like properties, emphasized by Wright and Granit (1938). But no red substances have ever been found in solution. The latter seem to be particularly elusive. There is, however, interesting work by Weigert and his collaborators [see bibliography (1940)] on visual purple dissolved in gelatin film which offers some promise, but we have perhaps not yet advanced far enough to make a discussion of his work profitable.

In one respect the experiments with microelectrodes have been disappointing. It has not been possible to ascribe with certainty any particular sensitivity band to a single active unit with the conviction that this unit delivers only one kind of message upwards. The experiments must count with the possibility of a given type of nerve fiber being able to deliver messages from different parts of the spectrum, depending also upon the state of adaptation. At the moment I do not think that color is determined by one-line connections for definite types of cones to the brain. And, indeed, if this were so why would the retina possess its very complicated switchboard of synaptic arrangements joining adjacent areas and twice interrupting the path upwards? At present the best explanation would seem to be that the perception of color is the response to integrated patterns caused by the interference in the retina of an assembly of simultaneously active sensitivity bands modulating a broad absorption curve of the type seen in frogs. This would probably be perceived as white and color could only be perceived when a sufficient number of modulators of a given type simultaneously are in operation thereby elaborating the pattern of impulses delivered upwards [cf. Wright and Granit (1938)]. On this view a small area or a low intensity would be incapable

of producing any other impression than that of white in proportion to the total discharge of impulses. This we know to be the case.

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Studies on the Characteristic Curve of Photographic Papers

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I. REFLECTION AND TRANSMISSION

IN this paper the term "reflection curve" will be used for the characteristic curve of photographic papers obtained by measuring the diffusely reflected light. The reflection density D being defined, as usual, by

$$D=-\log A,$$

A meaning the albedo of the paper (the albedo of an unexposed developed and fixed sample will be taken as unity).

The term "transmission curve" will be used for the H and D curve obtained with transmitted light, the transmission being defined, as usual, by

$$S=-\log T,$$

where T means the transparency (the transparency of an unexposed developed and fixed sample is taken as unity). Some details concerning the apertures and angles of incidence and reflection are indicated in Fig. 1.

The relation between transmission and reflection has been investigated repeatedly.

As far as we know the first formula was given by Mees, Nutting, and Jones[1]

$$D=\log 1/[(1-C)T^2+C],\ C \text{ being a constant.} \quad (1)$$

The first measurements were made by F. F. Renwick[2] who criticized sharply formula (1) and who presented another formula in a very thorough investigation with Channon and Storr.[3] Renwick's formula accounts for the diffuse reflection of the silver deposit and for the "interreflections." An analogous derivation, differing but formally from Renwick's, has been given more recently by Neugebauer.[4]

In applying his results to actual photographic material, however, Renwick made use of some approximations with the effect that the form of his final results differs from (1) virtually only in the meaning of the constants. Although Renwick's formula represents also only a very rough approximation, it will be sufficient for our further purposes to derive an analogous formula. We thereby introduce from the very beginning the factors which Renwick neglected.

Let T be the transparency of the layer, R the reflection power of its silver deposit. Then, if interreflections are disregarded, the albedo of the paper coated with the layer will be

$$A=T^n+R, \quad (2)$$

where n depends essentially upon the angles of incidence.

The disregard for the interreflections is justified if there is no optical contact between the emulsion layer and the reflecting paper. If such a contact is established, however, the total reflection at the interface between gelatin and air of a large part of the light diffusely reflected by the paper support plays an important role. For densities larger than 0.2 this effect can be accounted for sufficiently by properly adjusting the value of n.

The behavior of D at low densities and the exact interpretation of n—which seems distinctly lower at low densities, especially if optical contact is established between the support and the emulsion layer—present interesting but rather complicated problems which we do not intend to treat here. R itself in formula (2),

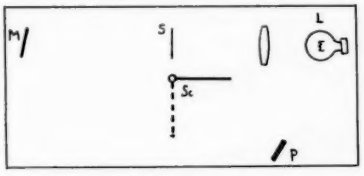

FIG. 1. Measuring device. L—light source. Dependent on the position of the screen sc the barrier layer photo-cell P measures either the light reflected diffusely by the sample s or the light transmitted by s and reflected on P by the mirror M.

[1] Jones, Nutting, and Mees, Phot. J. **38**, 342 (1914).
[2] F. F. Renwick, *Eders Jahrbuch* (1912), p. 106; Phot. J. **37**, 127 (1913).
[3] Channon, Renwick, and Storr, Phot. J. **42**, 121, 140 (1918).
[4] H. E. F. Neugebauer, Zeits. f. tech. Physik **18**, 137 (1937).

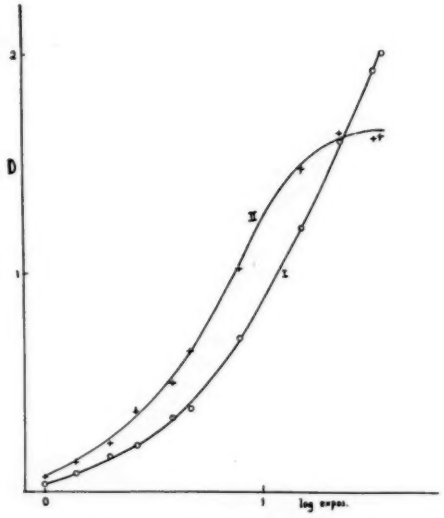

FIG. 2. Influence of silver-reflection. ○ Transmission densities; + reflection densities. II calculated according to formula (2) from curve I and the values of R which were measured separately.

however, obviously depends on T. Let dz be the number of grains in an infinitesimal thin layer of the coating; let z be the number of grains between this layer and the surface; let $T(z)$ represent the transparency of this same part of the coating; then the thin layer contributes to R:

$$dR=\rho T_{(z)}^{n}\cdot dz, \qquad (2.1)$$

ρ being a constant which characterizes the reflecting power of the grains. Disregarding once more the interreflections we put $T(z)=e^{-\epsilon z}$ and by integration of (2.1)

$$R=(\epsilon/\rho)[1-\exp(-\epsilon nZ)], \qquad (2.2)$$

where Z means the total number of grains per cm² of emulsion and $\exp(-\epsilon nZ)$ means the transparency of the whole layer. Let $\epsilon/\rho=R_0$ be a constant, dependent only upon properties of the individual silver grain. We rewrite

$$R=R_0(1-T^n) \qquad (3)$$

and by combining this with (2) we get

$$A=R_0+(1-R_0)T^n \qquad (4)$$

and

$$D=-\log[R_0+(1-R_0)\times 10^{-ns}]. \qquad (4.1)$$

With $n=2$ this corresponds exactly to Eq. (1). In this deduction the light diffusely reflected by the surface of the layer is not accounted for; it plays no role in most of the following considerations.

In Renwick's equation the mass of the silver deposit is used instead of s. In checking his equation, Renwick replaces the mass by the density as measured between opal glasses. He justifies this by referring to previous measurements.[5] These data, however, are in very poor agreement with Renwick's own statement. Moreover, according to our derivation, the measuring of the transmission densities has to be carried out under the same circumstances as the reflection density measurements. These, however, cannot easily be carried out with completely diffused light. We present, therefore, the results of some measurements of our own before further using Eq. (4). They were made with the simple device shown in Fig. 1.* We checked our derivation step by step. First formula (2) was tested by using a paper emulsion coated on glass. Its reflection was measured with a white paper support (result A) and without any support, i.e., with a "blackbody" behind the layer (result R); at the same time the transparency (T) was measured. Figure 2 shows that the simple relation (2) is sufficiently exact within the limits mentioned above.

Then relation (3) which relates the reflection of the scattering silver deposit to its transmission density was checked by comparing the R and T values. Figure 3 shows the results.

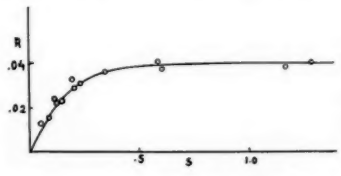

FIG. 3. Silver-reflection R as a function of transmission density. The line corresponds to formula (3).

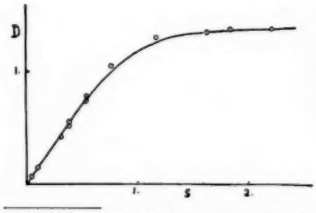

FIG. 4. Reflection density as a function of transmission density. The line corresponds to formula (4.1).

[5] F. F. Renwick, Phot. J. **37**, 203 (1913).

* Of course, the obvious corrections due to the imperfections of such an arrangement were made.

Finally Eq. (4) was checked by using a commercial photographic paper with even and thin support (Fig. 4). Equation (4) satisfactorily represents the experiments within the limits of error. Some more series were made with virtually the same results.

II. The Inflection Point and a Rule of Renwick's

The properties of an emulsion are usually judged by its H and D curve obtained with transmitted light: the transmission curve. The Eq. (4.1) shows in which way the characteristic curve of a paper (the reflection curve) is determined by the transmission curve of the emulsion on one hand, and on the other hand by the special conditions under which it is used, especially by the reflecting power of the silver deposit; the magnitudes s and D in (4.1) are to be read as functions of the log of exposure.

It is of some interest to see what conditions the transmission curve has to fulfill in order to yield certain properties of the paper curve. In order to obtain, e.g., a straight line portion in the paper curve (which may or may not be of any appreciable length) the transmission curve obviously has to be bent upward (convex toward the log E axis) in order to compensate for the effect of the silver reflection. Mathematically, this is easily shown by deriving from Eq. (4) the equation of the transmission curve which would yield a straight line reflection curve. Using the definitions

$$L=\log E, \quad G=\frac{dD}{dL}, \quad g=\frac{ds}{dL}, \tag{5}$$

where E means the exposure, and assuming G to be constant, we find from (4)

$$ds/dL=G/n(1-R_0 10^{-GL}) \tag{6}$$

as the differential equation of said transmission curve. With $G=1$ and proper adjustment of the integration constant, the solution can be written as

$$s=\frac{1}{n}\log\frac{1-R_0}{10^{-L}-R_0}. \tag{6.1}$$

The graph Fig. 5 shows this curve. A transmission curve as closely as possible resembling curve 3 of this diagram would be required in order to make a paper with a long straight line portion.*

The transmission curve of such a paper would have two approximately straight line portions, the one at low densities, the other at the inflection point. It is obvious that such a paper could be made, e.g., by a mixture of two different emulsions: one, with a small toe and a rather long straight line portion, would have to be used in such a quantity as to yield only a relatively low Γ and possibly a low maximum density; the other, much less sensitive emulsion should have a very high Γ and a very distinct "toe."

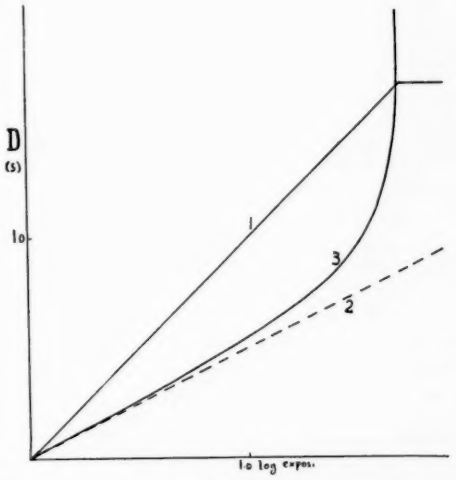

Fig. 5. The transmission curve corresponding to a straight line reflection curve. In order to obtain a straight line reflection curve (1) a transmission curve of the form (3) would be required. (2) shows the transmission curve which would yield curve (1) if no silver reflection occurred. The difference between (2) and (3) illustrates the influence of the silver reflection.

Whichever the form of the paper curve may be, obviously the transmission curve must be convex toward the log E axis at least up to the point which corresponds to the inflection point of the reflection curve (which practically represents a short straight line portion). The inflection point of the transmission curve, therefore, will be situated beyond this point. Since the inflection

* In this connection it is interesting to note that the straight line portion has virtually been abandoned in the Eastman Kodak papers, as Jones has recently stated. In view of the results obtained, however, e.g., with the chromate gelatin processes, papers of this type might still be of considerable interest for some purposes.

point of the paper curve is situated at rather high densities with all papers which are in practical use, by far the largest part, if not the whole, of the characteristic curve of the paper will correspond to the "toe" of the transmission curve. This has been stated long ago as an empirical rule by Renwick: "The gradation of silver printing papers is due to the under-exposure period of their emulsions." [6]

Though the gradient of the transmission curve in most of the cases shows some increase beyond the inflection point of the paper curve, as has just been pointed out, this increase ordinarily will not be made very large, since it would have no strong effect upon the behavior of the paper curve in the important shoulder region; in this region no practically obtainable steepness of the transmission curve is able to make up for the increasing effect of the silver reflection.

III. The "Shoulder" of the Reflection Curve and a Rule of Jones's

In his thorough investigation of the contrast of printing papers Jones[7] gave as a by-product a thumb-rule concerning the upper part of the reflection curve; the density D_b at the upper point b, where $G=0.2$, is situated about 0.02 below the maximum density:

$$D_{\max}-D_b=\Delta\doteq 0.02. \tag{7}$$

The figures in this purely empirical rule seem to be purely accidental. The facts which were discussed in the preceding part make possible, however, some approximate general statements concerning the so-called shoulder region. Using the definition (5) we get from (4)

$$R_0/A=1-G/ng. \tag{4.2}$$

At the "critical" point b the log R_0/A_b means the difference $D_{\max}-D_b$. Since R_0/A_b is certainly not far from unity, we find

$$\Delta=0.43G_b/ng_b. \tag{4.3}$$

Of course, we generally do not know the value of g corresponding to the point b; but we do know that in almost every practical case it will not be far from the G that corresponds to the inflection point of the paper curve, though presumably a little larger. Thus we may—as a first approximation—write

$$ng=\Gamma/(1-R_0/A^*), \tag{4.4}$$

where Γ means the maximum value of G and A^* the albedo at the inflection point (or rather at the upper end of the straight line portion). Since, as we mentioned already, the inflection point is usually not far from the shoulder, the value R_0/A^* is mostly of the order of $\frac{1}{2}$, and, as a very rough approximation, we may put $ng=2\Gamma$, keeping in mind that this represents rather a lower limit. Thus we get

$$\Delta=0.43G_b/2\Gamma\doteq 0.04/\Gamma. \tag{8}$$

With 37 types of papers investigated by Jones, the average value of Γ is about 1.7; so the average value of Δ should be near $\Delta\doteq 0.025$.

This is virtually the rule of Jones. We should expect, however, that this value might be rather too large. In fact, the average from Jones's data is about 0.015. In any case, our considerations enable us to understand well the order of magnitude of Jones's constant.

Formula (8), however, suggests a modification of that rule. While Jones states one approximate value of Δ, whatever the constants of the emulsion might be, our equation indicates that the point b should be nearer the maximum density with emulsions of more contrast.

The material of Jones enables us to check the relation (8) although only very roughly,* by plotting the available values (or average values) of $G_{\max}$ against the corresponding Δ: The result, represented by Fig. 6, seems to show that the modification of Jones's rule, suggested by the theory, means a distinct improvement.

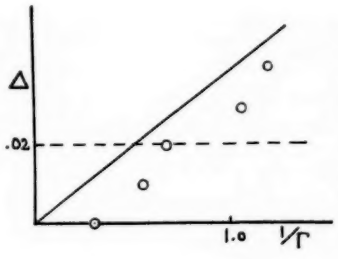

Fig. 6. "The upper limiting point." Distance of the upper limiting point from the maximum density as a function of Γ. The points are average values from Jones's work. The dotted line represents Jones's rule, the solid line formula (8).

[6] F. F. Renwick, Phot. J. **37**, 127 (1913).
[7] L. A. Jones, J. Frank. Inst. **202**, 177 (1926); **203**, 111 (1927); **204**, 41 (1927).

* The values of Δ are given but roughly and the values of $G_{\max}$ fluctuate widely with constant Δ.

The critical point b, discussed above, is important for the definition of an approximative measure of contrast as will be shown in the following part.

IV. The Measure of Contrast

Numerous attempts have been made to define a measure for the contrast of papers in a way meeting the practical requirements. In the authors' opinion, however, none of these attempts is better based on psycho-physiological considerations and none better checked by practical tests than the measure given by Jones in his classical paper.[7]

Recently Boutenbal[8] compared virtually all definitions given by different authors with a very extended material of practical tests, using various commercial papers. He finds five of the formulae —including that of Jones— to be in very good agreement with his statistical data. Three of them, however, are the average gradients or exposure scales in a range, the upper limit of which is given by $D_{max}-0.1$. If these measures were generally accepted, it would be very easy to produce papers of "high contrast," simply by means providing for a very low value D_{max}. (This criticism is, to a certain extent, analogous to Jones's well-known argument against the DIN sensitivity measure.) The fourth measure, the one proposed by Reinders and Boutenbal, is the exposure scale between $D=0.25$ and $D=0.9$. It is, however, obviously not desirable to make the judgment of contrast quite independent from the highlights and the deep shadows. Only the fact that all the papers used by Boutenbal had rather well-balanced characteristic curves, prevented disagreement of this measure with the judging of the prints.

Jones' formula, the fifth, being in good agreement with Boutenbal's test, is in no way met by that kind of criticism. It belongs, however, to those which, in Mr. Boutenbal's opinion, "require too great an amount of work for determining the contrast value." In fact, the determining of the contrast value according to Jones's rigorous definition involves an enormous amount of work. It will be shown below how this difficulty can be overcome, owing to some approximate relations between the magnitudes involved. The measure of contrast according to Jones is: the product of the numbers of just perceptible differences included in the density scale, by the average number of just perceptible differences per unit difference in log E. This means, practically, the product of the density scale by the average gradient of the paper, within that part of the characteristic curve that is actually used in making pictures. The determination of the actually used part of the scale, however, as well as that of the average gradient within given limits, is a very complicated procedure, if it is to be made in accordance with the strict requirements of Jones's psycho-physiological considerations.

Jones has shown that, in order to find the actually used part of the density curve, a statistical method is necessary, using several test negatives, a series of copies, and the opinion of several experts. The limiting points and the ranges of exposure and density determined in this way are widely different from those fixed by any simple rule, like the postulate that $G=0.2$.

The determination of the average gradient, according to Jones, has to be made with respect to equal increments of densities, not, as usually, with respect to equal increments of log exposure. Jones has shown that this correct $\bar{G}(D)$ differs widely from $\bar{G}(L)$.

Keeping in view, however, the S-like shape of the H and D curves and the location of the limiting points at its "toe" and its "shoulder," it is easy to see that a shifting of the limiting points influences the density scale and the $\bar{G}(D)$ values by the same order of magnitude but in the opposite direction. Although the postulate $G=0.2$ for the limiting points yields quite wrong values of the density scales as well as of the average gradient, the product of these values is affected but little by these errors. This statement can be substantiated by data from Jones's original paper, where all the values discussed here are to be found. Moreover, a remark in a more recent publication[9] of Jones and Morrison shows that the Kodak Laboratories use practically the limiting points $G=0.2$ with quite satisfactory results.

[8] C. Boutenbal, Phot. J. **78**, 76 (1938).

[9] L. A. Jones, J. Frank. Inst. **228**, 445, 605, 755 (1939).

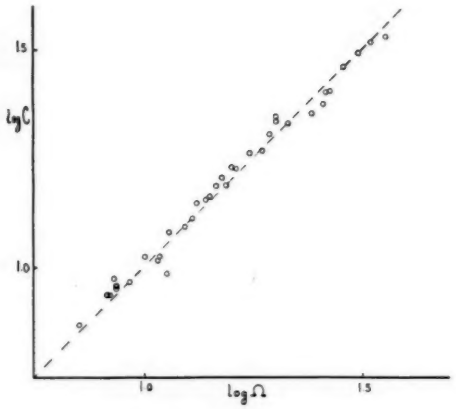

FIG. 7. Comparison of the values of contrast according to formula (9) with Jones's values (logarithmic scales).

There remains, however, the difficulty of calculating correctly $\bar{G}(D)$. This requires unusually exact determination of the H and D curve and its graphical differentiation (or a special equipment) and then some computation and graphical integration. In spite of some interesting simplifications found by Jones, quite a lot of work remains. This might be avoided in the following way: If $\bar{G}(D)$ is replaced by the usual average gradient $\bar{G}(L)$ with respect to equal increments of $\log E$—which is simply the quotient of the density scale over the exposure scale—it is obvious that much more stress is laid on the flat parts of the transmission curve than on the value $\bar{G}(D)$. Just the opposite error would be made by replacing $\bar{G}(D)$ simply by the maximum value of the gradient, Γ. This suggests the replacement of $\bar{G}(D)$ by a mean value of both, e.g., $(\Gamma \times \bar{G}(L))^{\frac{1}{2}}$. Using Jones's data, this was found to be feasible. Thus, on the basis of our considerations, we are led to try the following formula as an approximation for Jones's measure of contrast.

$$C = (\sigma^3 \Gamma / \lambda)^{\frac{1}{2}}, \qquad (9)$$

where σ is the density scale, λ the logarithmic exposure scale, both between the points where $G = 0.2$. The value of contrast according to this formula can be read from any H and D curve with reasonable accuracy. The major source of error is involved in the determination of the limiting points where $G = 0.2$. It was, however, mentioned already as a basis of the deduction of this formula that a shift in the limiting point does not matter very much. We formulate this now by differentiation of formula (9); taking into account that, at the points in question, $d\sigma = 0.2 d\lambda$, we get

$$dc/c = \tfrac{1}{2} \times (0.6/\bar{G}(L) - 1) d\lambda/\lambda. \qquad (9.1)$$

Using Jones's data for $\bar{G}(L)$ it is found that the relative error in the determination of the contrast due to the error in the limiting points is on the average only $\frac{1}{6}$ of the relative error of λ (even in the most unfavorable case found in Jones' material this error does not exceed $\frac{1}{3}$ that of λ).

Figure 7 shows the contrast as determined by formula (9) as a function of the contrast evaluated according to Jones's strict definition. The sequence of Jones's 37 types of papers with respect to their contrast is practically the same with both measures. The correlation coefficient is found to be 0.997, the few differences in the rank order are of no significance at all.

In only three cases does the change of C necessary to make both orders identical seem to exceed the limits of error, and in no case does it exceed 4 percent of the total range covered by the C values. In order to appreciate this agreement, let Jones's measures be compared with that represented by $\bar{G}(L)$. In this case the maximum change of C necessary to "restore the order" is about 25 percent. If the contrast values are grouped, as usual, into 5 or 6 classes, this error exceeds the whole range of at least one of them. With the measure given by formula (9) this could happen only if 25 or more classes of contrast were to be established.

It may be mentioned that formula (9) gives not only the same rank order as Jones's measure, but can be regarded as a good numerical approximation, as shown by the dotted line in Fig. 7. Jones has proposed, more recently, the use of the logarithm of his measure as a characteristic of contrast. This can be done, of course, with our approximation with practically equivalent results. The authors would consider the possibility of using the square root of C for the same purpose. This is suggested by the fact that, according to Jones's definition, his measure contains a product of two reciprocal values of least perceptible differences. Any monotonically increasing function of a measure of the kind in

question may be chosen to fit best the practical need for conveniently spaced values. The essential meaning of such a formula, however, is not expressed by those values, but by the rank order it establishes for its objects, which is not affected by such numerical changes.

Formula (9), which we propose as a practical basis for the rating of contrast, uses just those magnitudes which many authors have felt to be all-important for the same purpose. While these magnitudes have already been combined in different simple ways, we propose a combination which seems, at first glance, rather odd, but which is justified, however, by its close relation to Jones's definition of contrast. While formula (9) is thus founded on Jones's psycho-physiological considerations and his comprehensive empirical material, and supported also by Boutenbal's statistical investigations, its evaluation seems to be simple enough to encourage its practical use.

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Central Notations for ISCC-NBS Color Names

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THE problem of developing color designations "sufficiently standardized to be acceptable and usable by science, sufficiently broad to be used by science, art, and industry, and sufficiently commonplace to be understood, at least in a general way, by the whole public" was presented to the Inter-Society Color Council at its first meeting in 1931 by the Revision Committee of the *U. S. Pharmacopoeia*.

A report in 1933 (*1*)[3] of the Council's Committee on Measurement and Specification, I. H. Godlove, Chairman, provided the basis which was followed in developing what is now known as the ISCC-NBS[4] System of Color Designation. Setting boundaries for these color names proved to be the greatest problem, but in 1939 the work was completed at the National Bureau of Standards with the aid of the American Pharmaceutical Association and the U. S. Pharmacopoeial Revision Committee who for this purpose jointly supported a research associateship at the National Bureau of Standards. The Inter-Society Color Council in 1939 formally approved by letter ballot[5] and recommended to the U. S. Pharmacopoeial Revision Committee the method as described in a report by Judd and Kelly (*2*). Delegates from the Optical Society of America were active in cooperating with those from other member bodies and with individual members in developing this method.

The method adopted is simple in principle. The terms light, medium, and dark designate decreasing degrees of lightness, the adverb

TABLE I. *Abbreviations for use with ISCC-NBS system of designating colors*

NOUN FORM OF HUE	ADJECTIVE FORM OF HUE	ADJECTIVE MODIFIERS
Pk pink	pk pinkish	lt light
R red	r reddish	dk dark
O orange	o orange	wk weak
Br brown	br brownish	str strong
Y yellow	y yellowish	mod moderate
Ol olive	ol olive	med medium
G green	g greenish	viv vivid
B blue	b bluish	
P purple	p purplish	
Wh white		ADVERB MODIFIER
Gr gray		
Bk black		v very

[1] Color Technologist, and Chairman of the ISCC committee to obtain central notations for ISCC-NBS names.

[2] Member of committee.

[3] Italic numbers in parentheses refer to literature cited, p. 591.

[4] Initials are for Inter-Society Color Council-National Bureau of Standards.

[5] Out of 27 votes received (33 possible), 26 were for adoption. Except for one association, and one ballot marked "not voting," there were at least two affirmative votes (out of a possible 3) from each association represented in the Council at the time the vote was taken. There were no negative votes.

TABLE II. *Munsell notations for central colors of ISCC-NBS color names.*

NAME	MUNSELL NOTATION OF CENTRAL COLOR
Pinks and Reds	
pinkish white	6.5R 9.0/1.0
pinkish gray	6.5R 7.5/1.0
reddish gray	8.5R 5.5/1.0
dark reddish gray	7.5R 3.5/1.0
reddish black	7.5R 1.5/1.0
pale purplish pink	4.5RP 8.7/3.0
light purplish pink	4.5RP 8.5/7.0
brilliant purplish pink	4.5RP 8.5/11.0
weak purplish pink	4.5RP 7.0/3.0
moderate purplish pink	4.5RP 7.0/7.0
strong purplish pink	4.5RP 7.0/11.0
pale pink	2.0R 8.7/2.8
light pink	3.0R 8.7/6.2
brilliant pink	0.5R 8.5/11.0
weak pink	3.5R 7.3/2.5
moderate pink	4.5R 7.3/5.7
strong pink	4.5R 7.8/9.7
vivid pink	6.0R 8.2/13.0
pale orange pink	2.5YR 9.0/2.0
light orange pink	2.0YR 9.0/4.8
weak orange pink	2.5YR 7.5/2.0
moderate orange pink	2.0YR 7.5/4.8
strong orange pink	1.5YR 8.5/9.0
pale purplish red	0.5R 6.0/4.0
light purplish red	0.5R 6.0/8.0
brilliant purplish red	0.5R 6.0/12.0
weak purplish red	0.5R 4.5/4.0
moderate purplish red	0.5R 4.5/7.0
strong purplish red	0.5R 4.5/11.0
vivid purplish red	0.5R 4.5/14.0
dusky purplish red	0.5R 3.0/4.0
dark purplish red	0.5R 3.0/7.0
deep purplish red	0.5R 3.0/11.0
very dusky purplish red	0.5R 1.5/4.0
very dark purplish red	0.5R 1.5/7.0
very deep purplish red	0.5R 1.5/11.0
pale red	8.5R 6.0/2.7
light red	5.0R 6.0/8.0
brilliant red	5.0R 6.0/12.0
weak red	8.5R 4.5/2.7
moderate red	5.0R 4.5/7.0
strong red	5.0R 4.5/11.0
vivid red	5.0R 4.5/14.0
dusky red	8.5R 3.0/2.6
dark red	5.0R 3.0/7.0
deep red	5.0R 2.5/11.0
very dusky red	8.5R 1.5/2.6
very dark red	5.0R 1.5/7.0
Oranges and Browns	
light brownish gray	8.5YR 5.5/1.0
brownish gray	7.5YR 3.5/1.0
brownish black	7.5YR 1.5/1.0
light reddish orange	9.0R 7.0/9.0
brilliant reddish orange	9.0R 7.0/13.0
weak reddish orange	9.5R 5.7/5.5
moderate reddish orange	9.0R 5.5/9.0
strong reddish orange	9.0R 5.5/13.0
vivid reddish orange	9.0R 5.5/16.0
dark reddish orange	9.0R 4.0/10.0
deep reddish orange	9.0R 4.0/13.0
very pale orange	9.0YR 8.5/2.0
pale orange	6.5YR 8.5/5.0
light orange	5.5YR 8.5/9.0
brilliant orange	5.5YR 8.5/13.0
weak orange	5.5YR 6.6/5.4
moderate orange	4.5YR 6.5/9.0
strong orange	4.5YR 6.5/13.0
vivid orange	4.5YR 6.2/16.0
dark orange	4.5YR 5.0/9.0
deep orange	4.5YR 5.0/13.0
pale yellowish orange	0.5Y 9.0/5.1
light yellowish orange	10.0YR 9.0/9.0
brilliant yellowish orange	10.0YR 9.0/13.0
weak yellowish orange	0.5Y 7.5/5.1
moderate yellowish orange	10.0YR 7.5/9.0
strong yellowish orange	10.0YR 7.5/13.0
vivid yellowish orange	10.0YR 7.5/16.0
dusky yellowish orange	10.0YR 6.0/6.0
dark yellowish orange	10.0YR 6.0/9.0
deep yellowish orange	10.0YR 6.0/13.0
pale reddish brown	1.5YR 5.0/4.0
weak reddish brown	1.5YR 3.5/4.0
moderate reddish brown	9.0R 3.5/7.0
strong reddish brown	9.0R 3.0/11.0
dusky reddish brown	1.5YR 1.5/4.0
dark reddish brown	9.0R 1.5/7.0
deep reddish brown	9.0R 1.5/11.0
very pale brown	9.0YR 7.0/2.0
pale brown	8.5YR 5.5/2.0
light brown	5.0YR 5.3/5.0
weak brown	8.5YR 3.5/2.0
moderate brown	5.0YR 3.5/5.0
strong brown	4.5YR 3.5/9.0
dusky brown	8.5YR 1.5/2.0
dark brown	5.0YR 1.5/5.0
deep brown	4.5YR 1.5/9.0
light yellowish brown	1.0Y 6.0/4.0
moderate yellowish brown	10.0YR 4.5/5.2
strong yellowish brown	10.0YR 4.5/9.0
dark yellowish brown	10.0YR 2.5/5.2
deep yellowish brown	10.0YR 2.5/9.0
light olive brown	4.0Y 5.0/5.0
moderate olive brown	4.0Y 3.5/5.0
dark olive brown	4.0Y 1.5/5.0
Yellows and Olives	
yellowish white	7.5Y 9.0/1.1
yellowish gray	7.5Y 7.5/1.1
light olive gray	9.5Y 5.5/1.1
olive gray	9.5Y 3.5/1.1
olive black	9.5Y 1.5/1.1
pale yellow	5.5Y 9.0/3.3
light yellow	4.5Y 9.0/7.0
brilliant yellow	4.5Y 9.0/11.0
weak yellow	5.5Y 7.5/3.3
moderate yellow	4.5Y 7.5/7.0
strong yellow	4.5Y 7.5/11.0
vivid yellow	4.5Y 7.5/14.0
dusky yellow	5.0Y 6.0/4.0
dark yellow	4.5Y 6.0/7.0
deep yellow	4.5Y 6.0/11.0
pale greenish yellow	9.0Y 9.0/4.0
light greenish yellow	9.0Y 9.0/7.0
brilliant greenish yellow	9.0Y 9.0/11.0
weak greenish yellow	9.0Y 7.5/4.0
moderate greenish yellow	9.0Y 7.5/7.0
strong greenish yellow	9.0Y 7.5/11.0
vivid greenish yellow	9.0Y 7.5/14.0
dusky greenish yellow	9.0Y 6.0/4.0
dark greenish yellow	9.0Y 6.0/7.0
deep greenish yellow	9.0Y 6.0/11.0
pale olive	6.0Y 5.5/2.5
light olive	8.5Y 5.0/5.0
weak olive	6.0Y 3.5/2.5
moderate olive	8.5Y 3.5/5.0
dusky olive	6.0Y 1.5/2.5
dark olive	8.5Y 1.5/5.0
Greens	
greenish white	5.5G 9.0/1.0
light greenish gray	5.5G 7.5/1.0
medium greenish gray	5.5G 5.5/1.0
dark greenish gray	5.5G 3.5/1.0
greenish black	5.5G 1.5/1.0
weak olive green	4.0GY 3.5/2.0
moderate olive green	4.0GY 3.3/4.7
strong olive green	4.0GY 3.0/9.0
dusky olive green	4.0GY 1.5/2.0
dark olive green	4.0GY 1.5/5.0
deep olive green	4.0GY 1.5/9.0
pale yellow green	4.0GY 8.5/2.5
light yellow green	4.0GY 8.5/7.0
brilliant yellow green	4.0GY 8.5/11.0
weak yellow green	4.0GY 6.5/2.5
moderate yellow green	4.0GY 6.5/7.0
strong yellow green	4.0GY 6.5/11.0
vivid yellow green	4.0GY 6.5/14.0
dusky yellow green	4.0GY 5.0/2.5
dark yellow green	4.0GY 4.5/7.0
deep yellow green	4.0GY 4.5/11.0
very pale yellowish green	9.5GY 9.0/4.0
very light yellowish green	9.5GY 9.0/7.0
pale yellowish green	9.5GY 7.5/4.0
light yellowish green	9.5GY 7.5/7.0
brilliant yellowish green	9.5GY 7.5/11.0
weak yellowish green	9.5GY 5.5/4.0
moderate yellowish green	9.5GY 5.5/7.0
strong yellowish green	9.5GY 5.5/11.0
vivid yellowish green	9.5GY 6.5/14.0
dark yellowish green	9.5GY 4.0/5.0
deep yellowish green	9.5GY 4.0/9.0
very dark yellowish green	9.5GY 2.5/5.0
very deep yellowish green	9.5GY 2.5/9.0
very pale green	2.5G 8.5/2.0
very light green	5.5G 8.5/5.2
very brilliant green	6.0G 8.5/9.0
pale green	2.5G 6.5/2.0
light green	5.5G 6.5/5.2
brilliant green	6.0G 6.5/9.0
weak green	2.5G 4.5/2.0
moderate green	5.5G 4.5/5.2
strong green	6.0G 4.5/9.0
vivid green	6.0G 5.5/12.0
dusky green	6.0G 3.0/2.0
dark green	5.5G 3.0/5.2
deep green	6.0G 2.5/9.0
very dusky green	6.0G 1.5/2.0
very dark green	5.5G 1.5/5.2
very light bluish green	10.0G 8.5/4.8
light bluish green	10.0G 6.5/4.8
brilliant bluish green	1.0BG 7.2/9.0
moderate bluish green	10.0G 4.5/4.8
strong bluish green	1.0BG 4.5/9.0
vivid bluish green	1.0BG 4.5/12.0
dark bluish green	10.0G 3.0/4.8
deep bluish green	1.0BG 2.5/9.0
very dark bluish green	10.0G 1.5/4.8
very pale blue green	2.0BG 8.5/2.0
very light blue green	5.5BG 8.5/5.0
pale blue green	2.0BG 6.5/2.0
light blue green	5.5BG 6.5/5.0
brilliant blue green	5.5BG 7.0/9.0

TABLE II.—*Continued.*

NAME	MUNSELL NOTATION OF CENTRAL COLOR
Greens—Cont.	
weak blue green	2.0BG 4.5/2.0
moderate blue green	5.5BG 4.5/5.0
strong blue green	5.5BG 4.5/9.0
vivid blue green	5.5BG 4.5/12.0
dusky blue green	8.0BG 3.0/2.0
dark blue green	5.5BG 3.0/5.0
deep blue green	5.5BG 2.5/9.0
very dusky blue green	8.0BG 1.5/2.0
very dark blue green	5.5BG 1.5/5.0
Blues	
bluish white	6.0B 9.0/1.0
light bluish gray	6.0B 7.5/1.0
medium bluish gray	6.0B 5.5/1.0
dark bluish gray	6.0B 3.5/1.0
bluish black	6.0B 1.5/1.0
very light greenish blue	1.0B 8.5/5.0
light greenish blue	1.0B 6.5/5.0
brilliant greenish blue	1.0B 7.0/9.0
moderate greenish blue	1.0B 4.5/5.0
strong greenish blue	1.0B 4.5/9.0
vivid greenish blue	1.0B 4.5/12.0
dark greenish blue	1.0B 3.0/5.0
deep greenish blue	1.0B 2.5/9.0
very dark greenish blue	1.0B 1.5/5.0
very pale blue	6.5B 8.5/2.9
very light blue	8.5B 8.5/7.0
very brilliant blue	8.5B 8.5/11.0
pale blue	6.5B 6.5/2.9
light blue	8.5B 6.5/7.0
brilliant blue	8.5B 6.5/11.0
weak blue	6.5B 4.5/2.9
moderate blue	8.5B 4.5/7.0
strong blue	8.5B 4.5/11.0
vivid blue	8.5B 4.5/14.0
dusky blue	8.0B 2.5/3.1
dark blue	8.5B 2.5/7.0
deep blue	8.5B 2.5/11.0
very pale purplish blue	6.0PB 8.0/2.6
very light purplish blue	5.5PB 7.5/7.0
very brilliant purplish blue	5.5PB 7.5/11.0
pale purplish blue	6.0PB 5.5/2.6
light purplish blue	5.5PB 5.5/7.0
brilliant purplish blue	5.5PB 5.5/11.0
weak purplish blue	6.0PB 3.5/2.6
moderate purplish blue	5.5PB 3.5/7.0
strong purplish blue	5.5PB 3.5/11.0
vivid purplish blue	5.5PB 4.5/15.0

NAME	MUNSELL NOTATION OF CENTRAL COLOR
Blues—Cont.	
dusky purplish blue	6.0PB 1.5/2.6
dark purplish blue	5.5PB 1.5/7.0
deep purplish blue	5.5PB 1.5/11.0
Purples	
purplish white	6.0P 9.0/1.0
light purplish gray	6.0P 7.5/1.0
medium purplish gray	0.5RP 5.5/1.0
dark purplish gray	0.5RP 3.5/1.0
purplish black	0.5RP 1.5/1.0
very pale bluish purple	10.0PB 8.0/4.0
very light bluish purple	10.0PB 7.7/7.0
very brilliant bluish purple	10.0PB 7.5/11.0
pale bluish purple	10.0PB 5.5/4.0
light bluish purple	10.0PB 5.5/7.0
brilliant bluish purple	10.0PB 5.5/11.0
weak bluish purple	10.0PB 3.5/4.0
moderate bluish purple	10.0PB 3.5/7.0
strong bluish purple	10.0PB 3.5/11.0
vivid bluish purple	10.0PB 4.5/14
dusky bluish purple	10.0PB 1.5/4.0
dark bluish purple	10.0PB 1.5/7.0
deep bluish purple	10.0PB 1.5/11.0
very pale purple	5.5P 8.5/2.6
very light purple	5.0P 8.5/7.0
pale purple	5.5P 6.5/2.6
light purple	5.0P 6.5/7.0
brilliant purple	5.0P 6.5/11.0
weak purple	5.5P 4.5/2.6
moderate purple	5.0P 4.5/7.0
strong purple	5.0P 4.5/11.0
vivid purple	5.0P 4.5/14.0
dusky purple	5.5P 3.0/2.6
dark purple	5.0P 3.0/7.0
deep purple	5.0P 3.0/11.0
very dusky purple	5.5P 1.5/2.6
very dark purple	5.0P 1.5/7.0
very deep purple	5.0P 1.5/11.0
very pale reddish purple	9.5P 8.5/4.0
very light reddish purple	9.5P 8.5/7.0
pale reddish purple	9.5P 6.5/4.0
light reddish purple	9.5P 6.5/7.0
brilliant reddish purple	9.5P 6.5/11.0
weak reddish purple	9.5P 4.5/4.0
moderate reddish purple	9.5P 4.5/7.0
strong reddish purple	9.5P 4.5/11.0
vivid reddish purple	9.5P 4.5/14.0
dusky reddish purple	9.5P 3.0/4.0
dark reddish purple	9.5P 3.0/7.0
deep reddish purple	9.5P 3.0/11.0

NAME	MUNSELL NOTATION OF CENTRAL COLOR
Purples—Cont.	
very dusky reddish purple	9.5P 1.5/4.0
very dark reddish purple	9.5P 1.5/7.0
very deep reddish purple	9.5P 1.5/11.0
pale red purple	6.5R P 6.0/2.7
light red purple	4.5RP 6.0/8.0
brilliant red purple	4.5RP 6.0/12.0
weak red purple	6.5RP 4.5/2.7
moderate red purple	4.5RP 4.5/7.0
strong red purple	4.5RP 4.5/10.0
vivid red purple	4.5RP 4.0/14.0
dusky red purple	6.5RP 3.0/2.7
dark red purple	4.5RP 3.0/7.0
deep red purple	4.5RP 3.0/11.0
very dusky red purple	6.5RP 1.5/2.7
very dark red purple	4.5RP 1.5/7.0
very deep red purple	4.5RP 1.5/11.0
Grays and Near Grays	
white	N 9.2/
light gray	N 7.5/
medium gray	N 5.5/
dark gray	N 3.5/
black	N 1.3/
pinkish white	6.5R 9.0/1.0
pinkish gray	6.5R 7.5/1.0
reddish gray	8.5R 5.5/1.0
dark reddish gray	7.5R 3.5/1.0
reddish black	7.5R 1.5/1.0
light brownish gray	8.5YR 5.5/1.0
brownish gray	7.5YR 3.5/1.0
brownish black	7.5YR 1.5/1.0
yellowish white	7.5Y 9.0/1.1
yellowish gray	7.5Y 7.5/1.1
light olive gray	9.5Y 5.5/1.1
olive gray	9.5Y 3.5/1.1
olive black	9.5Y 1.5/1.1
greenish white	5.5G 9.0/1.0
light greenish gray	5.5G 7.5/1.0
medium greenish gray	5.5G 5.5/1.0
dark greenish gray	5.5G 3.5/1.0
greenish black	5.5G 1.5/1.0
bluish white	6.0B 9.0/1.0
light bluish gray	6.0B 7.5/1.0
medium bluish gray	6.0B 5.5/1.0
dark bluish gray	6.0B 3.5/1.0
bluish black	6.0B 1.5/1.0
purplish white	6.0P 9.0/1.0
light purplish gray	6.0P 7.5/1.0
medium purplish gray	0.5RP 5.5/1.0
dark purplish gray	0.5RP 3.5/1.0
purplish black	0.5RP 1.5/1.0

"very" is added to extend the lightness scale to "very light" and "very dark." The adjectives weak, medium, strong, vivid, designate increasing degrees of saturation. In order that unwieldy adjective combinations shall not be required, the following terms have been substituted for certain descriptions:

pale as a substitute for light, weak
brilliant as a substitute for light, strong
moderate as a substitute for medium, medium
dusky as a substitute for dark, weak
deep as a substitute for dark, strong

The relationship of these adjective modifiers is shown in Fig. 1. These terms are used to modify the following hue or limited hue range names: pink, red, orange, brown, yellow, olive, green, blue, purple, and combinations of these terms formed by using two of them, as in blue green, or an -ish suffix with one, as in purplish pink. For the neutral series the terms, white, gray, and black are used.

Certain of the hue names cover all lightnesses and saturations, as in the case of green. But

very pale (very light, weak)	very light	very brilliant (very light, strong)	
pale (light, weak)	light	brilliant (light, strong)	
weak	moderate (medium lightness, medium saturation)	strong	vivid (very strong)
dusky (dark, weak)	dark	deep (dark, strong)	
very dusky (very dark, weak)	very dark	very deep (very dark, strong)	

FIG. 1. Relation of adjective modifiers.

certain other hue names cover only a limited range of lightnesses and saturations, as the pinks which refer to very light reds, and the olives which refer to very dark yellows. But common usage is so strong in regard to such terms that the ISCC-NBS method has taken them into consideration and has applied different names to limited ranges of lightnesses and saturations of a single hue. Similarly, colors of yellow-red hue have been designated by three different hue names. The name "brown" is given to the dark group of colors, the term "orange" to those that are light and medium-to-strong in saturation, and the term "orange-pink" to those for light and weak yellow-reds. Each of the terms, brown, orange, and orange-pink is then divided into several designations by the use of appropriate modifiers selected from those shown in Fig. 1.

The color designations of the ISCC-NBS system define a block in the surface-color solid. The complete system provides for 312 such blocks, a number quite sufficient for many purposes, although it means that each block contains many distinguishable colors.

The boundaries of each designation are given in the 34 charts included in the Judd-Kelly report. For simplicity and ease of reference the limits were reported in terms of the Munsell color notation. It may be desirable ultimately to supplement this practical definition of limits for each color designation by giving equivalent definitions in terms of the standard coordinate system internationally adopted in 1931, since the standard system does not depend upon a material set of standards, as does the Munsell system. Meanwhile, interpolation curves made available by the laboratories of the Agricultural Marketing Service[6] already permit a fairly reliable transfer from Munsell notation to ICI specification, and it is expected that conversion tables of the colors in the Maerz and Paul *Dictionary of Color* will soon be available in Munsell notation. This will enable the standard ISCC-NBS color name for any color to be found by reference to the ICI, Maerz and Paul *Dictionary*, or the Munsell system directly, through the use of published conversion tables.

Because abbreviations probably will be used more often than not for the ISCC-NBS color designations, a notation for recording them is given in Table I. Capital letters refer to the noun form of the hue name; lower case letters are used for the adjective form. Since blue, brown, and black all begin with B, they are distinguished by writing the abbreviation B for blue, Br for brown, Bk for black. Green and gray are distinguished by using G for green, Gr for gray. Orange and olive are distinguished by using O for orange, Ol for olive.

The method of designation has been described before a number of the national associations that are member bodies of the Inter-Society Color Council (*3*)[7] and subcommittees of the Council have continued to work on various problems connected with the adoption of these color designations for, although they were officially adopted only for use with drugs and chemicals, other applications are being studied in the expectation that they will be adopted for more general use. The ISCC-NBS color designations are already being used in the new edition of the *National Formulary* and *U. S. Pharmacopoeia*, but it will take several years before changes are

[6] Dorothy Nickerson, "Use of ICI tristimulus values in disk colorimetry." Mimeographed preliminary report (1938). U. S. Department of Agriculture.

[7] In October, 1938, before the Optical Society of America (by D. B. Judd); in February, 1940, before the U. S. Pharmacopoeial Convention and National Formulary (by K. L. Kelly); in February, 1940, before the Technical Association of the Pulp and Paper Industry (by D. B. Judd); in March, 1940, before D-13 of the American Society for Testing Materials (by D. Nickerson); in April, 1940, before the Society of Motion Picture Engineers (by H. P. Gage); in September, 1940, before the Illuminating Engineering Society (by D. B. Judd); and in March, 1941, before the American Ceramic Society (by C. Robertson).

made in all the monographs, since the work of measuring all the drugs and chemicals named in them is considerable.

Early in 1941 a committee on the specification and production of central colors for the ISCC-NBS color designations was appointed (*4*).[8] Its first problem, assigned to the authors, was to obtain central notations for the color names for which boundaries had been set by Judd and Kelly, for not until these central notations are known can charts be made up to illustrate the ISCC-NBS system of names, and although these names are gradually coming into use, such use cannot be widespread until charts are available to illustrate them.

As might be expected, each central notation represents the approximate center of the corresponding block in the color solid. While some of these blocks are irregular in form, nearly always they could be divided into regular slabs. This made it possible to secure a central notation quite readily by making average determinations of hue, chroma, and value, weighted in accordance with the supporting volumes.

In Table II there are presented for the first time Munsell notations for the ISCC-NBS central colors. Use is already being made of these notations by the publication of color charts for specifying soil colors according to the ISCC-NBS system of names (*5*).

[8] Blanche R. Bellamy, Carl E. Foss, Walter C. Granville, S. M. Newhall, and Dorothy Nickerson, Chairman.

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(*3*) D. B. Judd and K. L. Kelly, "Scientific naming of drugs," J. Am. Pharm. Assn. **27**, No. 3 (March, 1938). D. B. Judd, "The Inter-Society Color Council Tentative System of Color Names," J. Opt. Soc. Am. **29**, 142A (1939). D. B. Judd, "Systematic color designations for paper," Tech. Assn. Pap. **23**, 512–518 (June, 1940), also Paper Trade J. (October 17, 1940), p. 35. Dorothy Nickerson, "Standardization of color names, the ISCC-NBS method," 1940 A.S.T.M. Standards on Textile Materials, D-13 Manual, also Am. Dyestuff Rep. **29**, No. 16, 392–396 (August 5, 1940). K. L. Kelly, "Instructions for determining color names for drugs and chemicals," Bull. Nat. Form. Comm. **8**, 11 (1940). H. P. Gage, "Color theories, and the Inter-Society Color Council," J. Soc. Mot. Pict. Eng. **35** (October 1940). D. B. Judd, "Color systems and their inter-relation," Illum. Eng. **36**, 3 (1941). C. Robertson, "Color standards for opaque ceramic materials," J. Am. Ceram. Soc. **24** (1941).

(*4*) Summary of first Committee Report, Minutes of Tenth Annual Meeting of the Inter-Society Color Council (March, 1941), p. 9.

(*5*) T. D. Rice, Dorothy Nickerson, A. M. O'Neal, and James Thorp, "Preliminary color standards and color names for soils," U. S. Department of Agriculture, Misc. Pub. No. 425. In press.

Photography by Luminescent Screens in the Extreme Ultraviolet

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Photography by luminescent screens for the extreme ultraviolet region λ1000~230A is described. A vacuum spectrograph with a one-meter concave grating at grazing-incidence was used to measure the response characteristics of various fluorescent materials. Luminescence photography by suitable materials should be of great aid in certain photometric measurements.

THE great improvement in the technique of extreme ultraviolet photography has been recently summarized by Boyce in his paper "Spectroscopy in the vacuum ultraviolet." [1]

The fact that luminescence is excited in zinc sulphide and zinc silicate by ultraviolet light was demonstrated by Rüttenauer in 1936.[2] Heyne and Pirani[3] coated a glass plate with fluorescent powders, placed it in front of a photographic plate in a quartz spectrograph, and thus obtained the spectrum between λ2600~2100A. Using a one-meter normal incidence vacuum spectrograph, Beese[4] photographed the spark spectra of nickel in the region λ2600~1000A, utilizing the luminescence of zinc silicate and calcium tungstate.

Schumann plates, now generally used, have been invaluable in exploring the extreme ultraviolet region of the spectrum. However, here we meet the difficulty that the Schumann plate was found unsuitable for photographic photometry.[5] Another method is to cover ordinary photographic plates with a thin coating of fluorescent mineral oils of various kinds[6] or with special sensitizers like naphthionic acid.[7] But the sensitivity of plates sensitized in this way gradually diminishes below λ1000A and practically ceases near λ500A.

Ilford "Q plates" have been found very useful for wave-lengths below λ300A;[8] these plates are, however, now very difficult to obtain.

With the increasing importance of intensity measurements, especially in the extreme ultraviolet region below λ1000A, some other procedure to obtain an accurate photographic photometry seems highly desirable.

The writers have recently found a method of luminescence photography which seems to be useful in the extreme ultraviolet region between λ1000A to 230A. As far as we know, luminescence photography has not been previously extended below λ1000A.

In studying luminescence photography, we have been using the same spectrographic apparatus referred to in a preceding paper.[9] A one-meter concave grating was used at grazing incidence. The light source was usually a hollow-cathode helium tube, as originated by Paschen, or a helium condensed-discharge tube. These sources were chosen because the former gives the ionized helium series beginning with λ303A, as well as the resonance series of 584A, while the latter gives a continuous spectrum between λ1000A to 600A.

To form the luminescent screen, a clear photographic film was smeared with a thin coating of the luminescent powder in the form of a suspension in water.

This luminescent film was mounted in the film holder in contact with, and in front of, a photographic film of high sensitivity. The incident extreme ultraviolet radiation stimulates the luminescent screen causing the film to blacken at the position of each spectrum line.

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[9] T. Suga, Sci. Pap. Inst. Phys. Chem. Research, Tokyo **34**, 7 (1937).

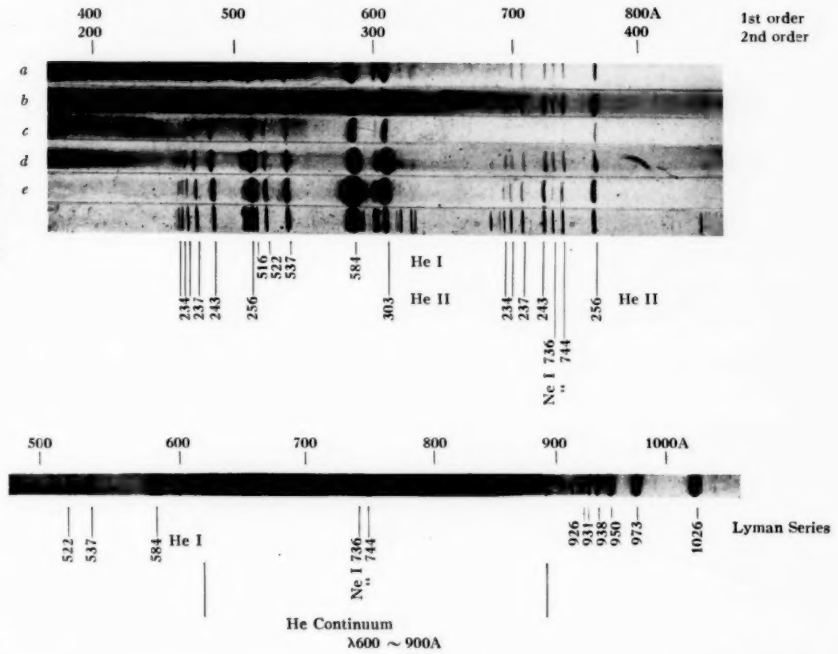

FIG. 1. Luminescent photograms of He lines. *a*—calcium tungstate; *b*—cadmium silicate; *c*—cadmium borate; *d*—magnesium tungstate; *e*—zinc silicate; *f*—Schumann film.

FIG. 2. Luminescent photogram (Zn silicate) of He continuum.

As luminescent powders we employed calcium tungstate, magnesium tungstate, cadmium silicate, zinc silicate, and cadmium borate, all from the Tokyo Electric Company.

It must be stated that satisfactory spectrograms by luminescent screens require exposures of 1 to 6 hours; by Schumann films, only 20 minutes to 3 hours.

In Fig. 1 we reproduce the spectrograms by luminescent screens of various kinds together with one on a Schumann film as comparison. We used a helium hollow-cathode tube as the source. The exposure of the luminescent photographs was 3 hours for each sample, and of the Schumann film 1 hour.

The first-order spectrum of the resonance helium lines and the second-order spectrum of the ionized helium lines can be observed on the same film enabling us quite conveniently to compare the relative response to the lines λ303A and λ584A.

In Fig. 1 it will be seen that the intensities of the fluorescent spectra are strong for cadmium silicate, magnesium tungstate and zinc silicate, and we can follow the ionized series down to the eighth or ninth member (λ230A).

In Fig. 2 we reproduce the spectrogram taken with an exposure of 3 hours, using a helium condensed-discharge tube. It may be mentioned here that the Lyman series of hydrogen and helium continuum come out quite strong.

The definition of luminescent photographs is naturally somewhat inferior to that of Schumann films, but perhaps we could improve it by using a luminescent screen of much finer grain.

Quantitative investigations of the above-mentioned luminescent screens have not yet been made. We believe, however, that even the qualitative results may be of some aid in certain photometric measurements in the extreme ultraviolet region.

In conclusion we wish to express our sincere thanks to Dr. T. Takamine for his kind guidance and the deep interest he has taken in the present experiments.

The Variation in Visual Acuity with Fixation-Distance

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(Received June 17, 1941)

IT has been shown by several investigations,[1,2] that the acuity of the human eye, as measured by the reciprocal of the size of the threshold stimulus, increases with fixation-distance under otherwise identical conditions. Since the pupil becomes smaller as the fixation-distance is decreased, the visual acuity would also be lowered due to the decrease in retinal illumination. However, a quantitative study of this factor indicates that, at most, it could account for a change of one percent in acuity. This is less than one-tenth of the observed change. Furthermore, the measurements of Cobb[3] have shown that over the usual range of pupil diameters the decrease in the size of the diffraction pattern apparently balances the increase in aberrations with increases in pupil size. It has also been postulated that the crystalline lens may be a more efficient refracting instrument when it is thin due to accommodation for distance than it is when it is relatively thick due to accommodation for near. Since the only ocular changes of optical significance are in the iris and lens, these two explanations must include, in some manner, the correct interpretation of the acuity changes. Recent data and analyses permit a more explicit statement as to the cause of the so-called acuity-distance phenomenon.

The present determination of the relationship between visual acuity and fixation-distance is based upon measurements of the threshold size of the Cobb[3] acuity test object. This object consists of two "white" bars on a "black" background which are separated by a distance equal to the width of the bars. Therefore, the width of the bars become the nominal dimension of the object. The actual test object is so constructed that the size of the parallel-bars is mechanically alterable without changing the geometrical proportions of the pattern. As used in the present investigation, the brightness contrast between the so-called white bars and their background was 92.5 percent, with the bars of the test object illuminated to a constant brightness of 10 footlamberts. All of the measurements were made in a room indirectly illuminated to about 15 footcandles, as measured in a horizontal plane at desk height.

The measurements were made by seven adult emmetropic subjects (with ophthalmic corrections where necessary) each of whom made five measurements on each of four different days for each of six fixation-distances varying from 0.5 to 10 meters. In alternate test periods, the series of measurements were made with increasing and decreasing fixation-distances, respectively. The measurements of the threshold of resolution were made binocularly, thus insuring, in this respect, a normal functioning of the accommodative mechanisms. The data are summarized in Table I.

It will be noted from Table I that the relationship between fixation-distance and the size of the threshold stimulus is a linear one which may be expressed with precision by the equation: $S=0.02+0.2173D$, where S is the threshold size in millimeters and D is the fixation-distance in meters. This equation was derived from the experimental data by graphical methods; and its

TABLE I. *These data are arithmetic means of the individual measurements of threshold size. The mean of the series of five measurements at each distance by each subject was used as a single value in computing the probable errors.*

Distance in Meters	Size of Object in mm	Probable Error mm	Probable Error Per-cent	Probable Error Per-cent (*)	Size of Object (Computed)	Visual Acuity
0.5	0.1291	0.0027	2.1	1.8	0.1286	1.13
1.0	0.2374	0.0041	1.7	1.6	0.2373	1.23
2.0	0.4568	0.0078	1.7	1.8	0.4546	1.28
5.0	1.091	0.014	1.3	1.2	1.106	1.32
8.0	1.758	0.026	1.5	1.1	1.758	1.33
10.0	2.263	0.032	1.4	1.2	2.193	1.33

* Excluding the systematic differences between subjects.

[1] Ellis Freeman, "Anomalies of visual acuity in relation to stimulus-distance," J. Opt. Soc. Am. **22**, 285 (1932).
[2] Matthew Luckiesh and Frank K. Moss, "The dependency of visual acuity upon stimulus distance," J. Opt. Soc. Am. **23**, 25 (1933).
[3] Matthew Luckiesh and Frank K. Moss, *The Science of Seeing* (D. Van Nostrand Company, New York, 1937), p. 492.

appropriateness may be estimated by comparing the data in the second and sixth columns of Table I. It will be noted that the average differential between the experimental and computed values is less than one percent. The empirical term of 0.02 mm may be considered to represent an increment in the size of the threshold stimulus required to compensate for the blurring of the retinal image by aberrations. That this term should be substantially independent of distance is consistent with theoretical considerations. Upon the plausible assumption that the acuity in angular measure is a maximum at distance and decreases by an amount proportional to the accommodation in *diopters*, it follows that $S/D=A+B/D$ where A is the angular threshold at distance, B is the proportionality constant for the increase in angular size and D is the fixation-distance in meters. Clearing of fractions, this becomes $S=B+AD$ which is the formula empirically found. Since such a constant is relatively more important in near-vision than in distant-vision, it follows that visual acuity would vary with fixation-distance. Thus the observed fact that the acuity is greater at distance than at near appears to result from a blurring of the retinal image due to aberrations which is of greater importance with small than with large *threshold* test objects. The supposition that the neuromuscular mechanisms of accommodation and convergence function more efficiently with fixation at distance than at near is also a possible explanation if it be assumed (cf. Weber's law) that the average error is a constant fraction of the accommodation required. In essence, the choice is between organic and functional aberration. It is entirely possible that both contribute to the observed changes in acuity. The linearity of the resultant aberration with the necessary accommodation is established by these experiments since the average disparity between the experimental data and those computed from the linear equation relating threshold size and fixation-distance is less than one percent.

The authors are indebted to Mr. A. A. Eastman of the Lighting Research Laboratory for obtaining the precise data which made possible the present analyses of the visual-acuity —stimulus-distance anomaly.

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The Refraction of Various Spectral Qualities of Light by the Human Eye

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(Received July 19, 1941)

THERE is a considerable diversity in the spectral character of illuminants used for lighting purposes, represented by natural sunlight and skylight, light from tungsten filament lamps and from the relatively new fluorescent lamps. As a consequence, a variety of questions arise pertaining to the differences in the refraction or focusing of these lights by the human eye. When normal eyes are converged upon an object illuminated by so-called white light, it is tacitly assumed that the image of the object is focused upon the retina. However, if the object is illuminated by light of a more restricted spectral range, it is obvious that the normal relationship between convergence and accommodation generally must be altered if the image is to remain in focus. For example, green light would be focused anterior to the retina (toward the cornea) if the convergence-accommodation relationship remained unchanged; and red light would be focused posterior to the retina. If the dioptric displacement of the focal plane is appreciable, this fact may be important in certain lighting situations involving the use of colored light which is now efficiently produced by fluorescent lamps. The purpose of the present investigation is to determine the magnitudes of the differences in focus for light provided by various types of fluorescent lamps.

The method of obtaining these data is similar to that employed in determining refractive errors of the eye. Thus an "error" in focus due to the spectral quality of light is analogous to a refractive error in the optical system of the eye. The prosecution of such a research was facilitated by the recent development of an unusually precise method of subjective refraction.[1] In this method, the refractive error of the eye is measured by the power of the lens, either plus or minus, which must be added to the ocular refractive system in order to cause a given achromatic test object to be maximally visible. Since the eye is capable of compensating for rather large refractive errors through the exercise of relative accommodation,* it is obvious that this function must be inhibited for the purposes of the present investigation. This is accomplished through the use of the Luckiesh-Moss ophthalmic sensitometer—an instrument and technique which permits the functioning of convergence in the proved absence of any adequate stimulus for either positive or negative relative accommodation.[1]

The refracting power of normal eyes under tungsten filament light and various qualities of light from fluorescent lamps was determined for fixation distances of 16 inches and 20 feet, respectively. Since these distances are usually considered appropriate for so-called near-vision and distant-vision tests, the data may be interpreted into seeing at any distance. Fifteen separate refractions were made under each quality of light with each of two adult emmetropic subjects with fixation at 16 inches; and five separate refractions were made under each experimental condition with fixation at 20 feet. The two subjects were selected from many whose vision has been studied with various clinical and laboratory techniques. Furthermore they were highly skilled in the use of the L-M sensitometer.

TABLE I. *The additional dioptric power required by two subjects possessing normal vision to produce maximal visibility, in the absence of relative accommodation, under four different spectral qualities of light assuming the refractive state indicated under tungsten filament light as a base. (The measurement of any lens in diopters is the reciprocal of its focal distance in meters, plus for convex and minus for concave lenses.)*

QUALITY OF LIGHT	SUBJ. *A*	SUBJ. *B*	AVERAGE
Added power in diopters with fixation at 16 inches			
Red fluorescent	+0.20	+0.21	+0.20
Tungsten filament	0.0	0.0	0.0
Blue fluorescent	−0.15	−0.32	−0.24
Green fluorescent	−0.30	−0.32	−0.31
Added power in diopters with fixation at 20 feet			
Red fluorescent	+0.04	+0.15	+0.10
Tungsten filament	0.0	0.0	0.0
Blue fluorescent	−0.25	−0.12	−0.18
Green fluorescent	−0.25	−0.22	−0.24

TABLE II. *The additional power required to produce maximal visibility as determined from single refractions of five subjects.*

QUALITY OF LIGHT	*A*	*B*	*C*	*D*	*E*	AV.
Added power in diopters with fixation at 16 inches						
Red fluorescent	+0.21	+0.19	+0.29	+0.34	+0.46	+0.30
Tungsten filament	0.0	0.0	0.0	0.0	0.0	0.0
Daylight fluorescent	−0.02	0.0	−0.01	−0.03	−0.10	−0.03
Blue fluorescent	−0.23	−0.22	−0.16	−0.08	−0.09	−0.16
Green fluorescent	−0.19	−0.29	−0.31	−0.23	−0.21	−0.25

The data are summarized in Table I. It will be noted that the refractive state of the eye under tungsten filament light is taken as a base for determining the dioptric displacement of the point of focus under lights of various spectral characters provided by fluorescent lamps. This assumption is in accord with ophthalmic practice. It is also justified on the basis that human eyes developed the so-called normal relationship between accommodation and convergence under light approximating tungsten filament light in spectral composition. The probable errors of the dioptric values given are of the order of two- or three-hundredths of a diopter.

These data reveal that rather extreme departures in spectral quality from that of tungsten filament light produce only small differences in focus. It is apparent from these data that the light from present fluorescent lamps is far from monochromatic in character since the chromatic difference in focus for the extremes of the visible spectrum is of the order of two diopters. Specifically, it will be noted from Table I (near vision) that red fluorescent light is focused about 0.2 diopter posterior to the retina on the basis that tungsten filament light is focused exactly upon

[1] Matthew Luckiesh and Frank K. Moss, "New method of subjective refraction involving identical technics in static and in dynamic tests," Arch. Ophthal. **23**, 941 (1940).

* Relative accommodation: a change in the focusing of the crystalline lenses relative to that which is normal for a given state of convergence of the two eyes. When the change is in the direction of greater convexity of the crystalline lens, it is designated as positive.

the retina by an emmetropic eye. Similarly, it will be noted that green fluorescent light is focused anterior to the retina by about 0.3 diopter. Thus the focal disparity between the red and green fluorescent lights is about 0.5 diopter. The data of Table I also indicate that the dioptric effects of alterations in spectral quality are somewhat less for distant vision than for near vision. However, the differences due to fixation distance appear to be negligible from various practical and clinical viewpoints. It is also of interest to note that the green fluorescent light is refracted to a greater degree than the blue fluorescent light. This result is an indication of the relatively low degree of saturation of the light from present-day blue fluorescent lamps. It was also adequately established by a few measurements that there was no significant difference in the focus of tungsten filament, white fluorescent and daylight fluorescent lights.

Although only two subjects were studied extensively in this investigation, the data obtained are considered reliable in view of the extremely close agreement between the subjects in the order of magnitude of the chromatic differences in focus. This conclusion is further supported by the fact that single refractions for five different subjects yielded average values in close agreement with those given in Table I as will be noted from Table II. In appraising the significance of the experimental data, it may be helpful to consider the fact that ophthalmic lenses are ground in steps of 0.125 diopter; and that many refractionists believe it is unnecessary to correct spherical errors of less than 0.25 diopter. The effect of such errors upon visibility,[2] if uncorrected, may be estimated from Fig. 1.

In nearly all situations of near vision, the mechanisms of relative accommodation are more than adequate to compensate for dioptric disparities of the order of those given in Table I.

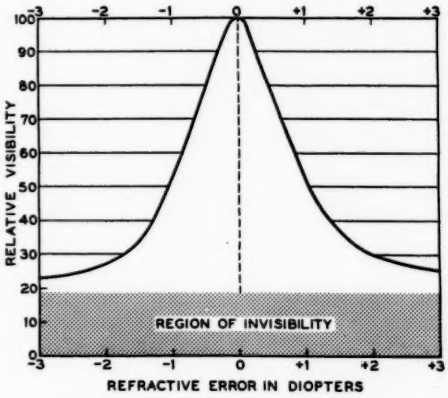

FIG. 1. The loss in visibility due to uncorrected refractive errors, as determined with the L-M Ophthalmic Sensitometer for a typical emmetropic subject. Visibility is expressed in terms of the visual factor of size. [Matthew Luckiesh and Frank K. Moss, "Visibility—its measurement and significance in seeing," J. Frank. Inst. **220**, 431 (1935).] For example, a relative visibility of 50 percent refers to an object one-half as large as a similar object having a relative visibility rating of 100 percent.

Hence objects viewed at close range could be focused on the retina under all fluorescent illuminants through the exercise of relative accommodation; that is, through a change in accommodation while the point of fixation (convergence) remains unchanged. Such a compensation for chromatic differences in focus is analogous to the compensation of low degrees of hyperopia (cf. red fluorescent) and myopia (cf. green fluorescent) by appropriate accommodative activity. Although such a compensation is readily effected by the normal eye, it might be undesirable if long continued. In distant vision a requirement of additional negative power of the order of 0.25 diopter may be a definite handicap from the viewpoint of visibility. Thus distant objects illuminated by green fluorescent or blue fluorescent light may not be seen distinctly due to the inability of the observer to supply the required negative power. Since, except in presbyopia, adequate positive accommodation is possible in distant-vision, objects illuminated by red fluorescent light could be focused readily.

[2] Matthew Luckiesh and Frank K. Moss, "Characteristics of sensitometric refraction," Arch. Ophthal. **25**, 576 (1941).

The Distribution of the Latent Image in the Silver Bromide Grain*

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In three different commercial emulsions, the distribution of the latent image between the surface and the interior of the silver bromide grains was investigated. The results show the sensitivity centers to be almost exclusively on the surface of the grains. Competitive centers are formed in the interior by strong exposures.

PREVIOUS FINDINGS

WITH the discovery of development centers existing inside the silver bromide grains (by W. Clark and other authors) the problem of their distribution presented itself. In an early investigation Noddack[1] showed that only 1/40 to 1/100 of the photolytic silver was found to be on the surface of the grains. In view of the close parallelism between the latent image and photolytic silver, it was assumed that only a very small part of the latent image could be developable, since only the centers at the surface are accessible to an ordinary chemical developer. By comparing the number of developable grains at the threshold of the density with the number of absorbed quanta, Eggert and Noddack[2] arrived at a ratio of the same order, 1 : 300, and they pointed out that this ratio, 1 : 300, also holds for the relation between the number of molecules at the surface and that in the whole volume of a spherical silver bromide grain of average size. If equal conditions for the formation of centers prevail throughout the grain, then their experimental result should be expected. This reasoning, however, does not take into account the pre-formation of sensitivity centers, unless it is assumed that the conditions for their formation are also equally favorable throughout the grain.

In 1937, a very extensive investigation of the distribution of photolytic silver was undertaken by Meidinger.[3] He established the fact that with increasing exposure the silver in the interior of the grains increases, but decreases relatively to the surface silver. But, since it is quite uncertain in what way this dependence might extend to lower exposures, the distribution of the latent image could not be extrapolated from the distribution of the photolytic silver.

A direct investigation of the latent-image distribution was published by Kempf.[4] After several film strips were given equal exposures, he subjected one strip to chemical development and in this way determined the density resulting from the surface of the grains. With the other strips he proceeded to the next layer inside the grains by first removing the latent image on the surface with an oxidizing agent, and then dissolving the outside of the grains by a short immersion in a fixing bath. In one of these strips the density was determined after chemical development, while the alternative treatment with oxidizing and fixing bath was repeated with the remaining strips, so that a second layer, further inside the grain, was uncovered. The whole procedure was repeated until the densities of four layers finally were obtained, i.e., of the surface and of three levels in increasing distances from the surface. The density was found to decrease from the surface to the interior, but the decrease was less than proportional to the decrease in the grain radius which was determined microscopically. This relative increase from the surface to the interior means that an even smaller part of the latent image was formed at the surface than would be expected, if it were equally distributed throughout the grain. This result was confirmed by the application of post-fixation physical development to the grains in their various stages, and by the analytical determination of the developed silver.

For two reasons this interesting result is not

* Communication No. 814 from the Kodak Research Laboratories.
[1] Cf. W. Leszynski, Zeits. f. wiss. Phot. **24**, 271 (1926).
[2] J. Eggert and W. Noddack, Naturwiss. **15**, 57 (1927).
[3] W. Meidinger, Physik. Zeits. **38**, 564, 737, 905 (1937).
[4] A. Kempf, Zeits. f. wiss. Phot. **36**, 235 (1937).

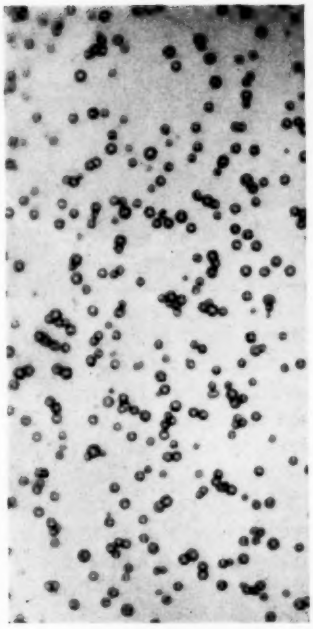

FIG. 1.

FIG. 2.

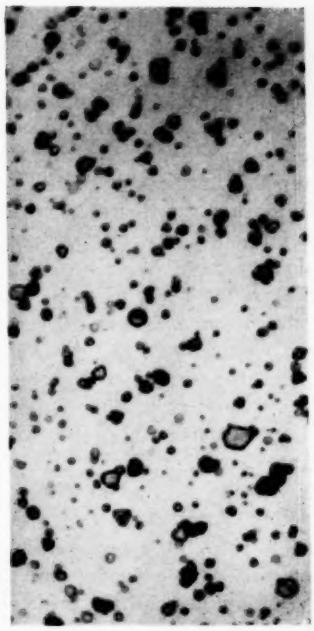

FIG. 3.

immediately applicable to photographic conditions. First, it is not known how the distribution might be found to vary with the exposure. Secondly, the emulsions under investigation could not be regarded as typical photographic emulsions. The grains of an ordinary photographic emulsion are too small and too irregular in shape to permit the accurate topographic study which was carried out by Kempf. He used special conditions and an inert gelatin in order to grow equal-sized grains 12 to 14 microns in diameter. These emulsions had a very low speed, and it is doubtful whether pre-formed sensitivity centers played any part at all in the formation of the latent image.

In order to gain information on latent-image distribution in commercial photographic emulsions, an experimental investigation of this distribution was carried out by the author with three different emulsions.

EXPERIMENTAL PROCEDURE

Since the grains of the usual photographic emulsions present great difficulties for an accurate topographic study of the latent image, the task was restricted to a comparison between the latent image on the surface and in the interior of the grains.

The use of chemical development seemed inadvisable for this purpose since only one layer in the interior could be determined in this way; with the density depending on the size of the remaining part of the grain, the comparison of the interior latent image between different emulsions cannot be well defined, since the speed of reaction in the fixing bath varies considerably from emulsion to emulsion.

With physical development, the latent image of the whole grain and of the grain without surface silver can be compared under identical conditions. This method has the additional advantage of giving direct information about the number of development centers, whereas chemical development provides information about the grains which have *at least* one development center accessible. The nearly linear dependence of the physically developed silver on the number of centers was established by Arens and Eggert.[5] At

[5] H. Arens and J. Eggert, Zeits. f. Elektrochemie **35**, 728 (1929).

the maximum of the density curve the centers inside the grain frequently coalesce during fixation to one large center and the developed silver is no longer proportional to the number of the original centers.[6] Ungar,[7] in criticizing Kempf's work, rejected physical development on these grounds. However, even if the method is not sufficiently accurate at the maximum, the objection can only apply to that region, whereas the main objective of this investigation was the initial part of the density curve, in order to gain definite information about the distribution of the sensitivity centers. Only at very low exposures can the pre-formed centers be expected to determine the distribution of the latent image; at higher exposures other centers can be formed by the action of light itself.

A method of post-fixation development designed to extend density measurements as far as possible into the region of low exposures was described in a previous paper.[8] The density curves obtained in this way are of no real significance for the emulsion, but it was shown that two exposed film strips treated in the same way gave practically identical results for each intensity step. Therefore, an exposed film, from which the surface silver had been dissolved, could be compared with an exposed film strip which had not undergone any treatment before the post-fixation development. In this way the latent image of the interior could be compared with the latent image of the whole grain.

Ammonium persulfate was chosen for the oxidizing bath. Kempf had shown that the removal of the surface image was complete after a 2-hour treatment in a $N/10$ solution of ammonium persulfate. Accordingly, the film strip was suspended for two hours in such a solution, with the emulsion side facing downwards. The results show that this treatment was sufficient with the films used in this case too; this was fortunate, since an increase in concentration or time would have been harmful. Even under the existing conditions, special hardening of the films was necessary in order to avoid undesirable effects on the gelatin. After the persulfate treatment, the film strip was thoroughly washed for 40 minutes and then, together with the comparison strip, for another 20 minutes before the treatment in the fixing bath. The densities of the developed strips were measured on a Capstaff-Purdy densitometer by means of a photronic cell.

TABLE I. *Exposure time—15 min. Emulsion I.*

Emulsion I, A			Emulsion I, B		
Whole Density	Interior Density	R_1	Whole Density	Interior Density	R_1
0.88	0.99	1	1.07	1.02	1
0.72	0.69	1	0.79	0.66	1
0.55	0.39	1	0.63	0.37	2
0.37	0.18	2	0.49	0.16	3
0.28	0.07	4	0.35	0.07	5
0.21	0.01	21	0.18	—	>18
0.14	—		0.06	—	
—	—		0.01	—	

TABLE II. *Exposure time —2 min. Emulsion II.*

Whole Density	Interior Density	R_1
0.34	0.18	2
0.26	0.12	2
0.20	0.11	2
0.18	0.06	3
0.17	0.04	4
0.14	0.02	7
0.12	—	
0.10	—	

TABLE III. *Exposure time —½ min. Emulsion III.*

Whole Density	Interior Density	R_1
1.14	0.16	7
0.94	0.14	7
0.70	0.09	8
0.48	0.08	6
0.31	—	
0.09	—	
0.07	—	
0.07	—	

The Results

Three different emulsions were investigated in this way: emulsion I, a low speed emulsion, consisting of rather uniform small grains (Fig. 1); emulsion II, of much higher sensitivity, consisting of fairly uniform large grains (Fig. 2); and emulsion III, of still higher sensitivity, consisting of grains of various sizes (Fig. 3). The recorded densities are all corrected for fog.

The results with emulsion I are given in Table I. Two similar experiments, A and B, are recorded in order to indicate the margin of error. The intensity steps varied with the factor two, and the exposure time was 15 minutes; R_1 is the ratio between the developed densities in the whole grain and those in the interior, and, therefore, the ratio for the latent image centers in the same regions. The figures show that at low exposures the overwhelming part of the latent image is on the surface of the grain; this indicates that the

[6] H. Arens, Zeits. f. wiss. Phot. **31**, 68 (1932).
[7] G. Ungar, Zeits. f. wiss. Phot. **37**, 130 (1938).
[8] G. Kornfeld, J. Frank. Inst. **229**, 181 (1940).

few sensitivity centers are entirely concentrated on the surface. With increasing exposures, the rate of formation of new centers in the interior increases rapidly, and the results at the highest exposures show how successfully these new centers compete with the surface centers—the entire latent image is formed in the interior.

Table II records an experiment with emulsion II. The same intensity steps were used, but the exposure time was only 2 minutes. These shorter exposures do not give such ample opportunity for the formation of interior centers, and the competition with the more numerous sensitivity centers at the surface is less keen. Accordingly, only at the highest exposures is parity reached between the interior and the surface. But even at the lowest exposures a small fraction of the latent image originates in the interior.

Results with emulsion III are recorded in Table III. The exposure time was 30 seconds and the intensity steps were unchanged. No centers seem to have been formed by these exposures; the ratio between the density in the whole grain and that in the interior does not vary. The very existence of latent-image centers in the interior is remarkable. They constitute about the same fraction of the total number as with emulsion II. This seems to indicate that with sensitive emulsions, a small fraction of the sensitivity centers are formed in the interior, even though crystal growth is practically completed before after-ripening.

In order to compare the results obtained with the three emulsions, the interior densities D_1, and the values for the ratio R_2, between surface density and interior density, are presented in Table IV at corresponding steps of time-intensity products. This comparison is only an extremely rough approximation, since it ignores not merely reciprocity law failure, but also the arbitrary character of the density curves. The ratio of the densities, however, will be less affected by this second factor, unless it reaches an extreme value. The actual increase of surface density at higher exposures may have an additional cause in the rebromination of the surface image recently investigated by Sheppard and Graham.[9]

Shortly before this work was completed, the author became acquainted with a very extensive investigation on latent-image distribution by Berg, Marriage, and Stevens.[10] They used a different method and they obtained very important results on various photographic effects. The increase of the latent image in the interior of the grains with increasing exposure is confirmed by this work.

TABLE IV. *Comparison of latent-image distribution.*

Emulsion I		Emulsion II		Emulsion III	
D_1	R_2	D_1	R_2	D_1	R_2
0.99	0.05				
0.69	0.2				
0.39	0.7				
0.18	2	0.18	1		
0.07	4	0.12	1		
0.01	>18	0.11	1	0.16	6
—		0.06	2	0.14	6
—		0.04	3	0.09	7
—		0.02	6	0.08	5
—		—		—	

ACKNOWLEDGMENT

The author feels very much indebted to Dr. B. H. Carroll of these Laboratories for the special preparation of emulsion films. She also expresses sincere thanks to Mr. R. P. Loveland of these Laboratories, in whose department the photomicrographs were taken.

[9] S. E. Sheppard and C. L. Graham, J. Frank. Inst. **230**, 619 (1940).

[10] W. F. Berg, A. Marriage, and G. W. W. Stevens, J. Opt. Soc. Am. **31**, 385 (1941).

Unit Planes of a Plane-Parallel Plate

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BY considering the plane-parallel plate as the limit of a simple lens with the radii of its surfaces numerically equal and converging to infinity, Pestrecov[1] has obtained two solutions for the limiting locations of the unit points. It is the purpose of this note to consider convergence of the radii to infinity in any manner whatever, and in particular to show that the location of the unit points may similarly be assigned to any pair of conjugate points of the plate.

Let V_1 and V_2 be the vertices of the lens surfaces, R_1 and R_2 the radii, H and H' the principal points, f the focal length, t the thickness, and n the index of the glass. Let s and s' be the object and image distance, respectively, measured from the principal points, and let u and v be object and image distances referred respectively to the first and second surfaces of the lens or plate. Distances are positive at the right and negative at the left of a reference point. It is assumed that the optical system is surrounded by air. The following well-known formulas[2] are taken as a basis:

$$1/f=(n-1)[1/R_1-1/R_2+(n-1)t/nR_1R_2]. \quad (1)$$

$$V_1H=-(n-1)tf/nR_2=tR_1/[n(R_1-R_2)-(n-1)t]. \quad (2)$$

$$V_2H'=-(n-1)tf/nR_1=tR_2/[n(R_1-R_2)-(n-1)t], \quad (3)$$

$$HH'=t-(R_1-R_2)t/[n(R_1-R_2)-(n-1)t], \quad (4)$$

$$1/s'=1/f+1/s. \quad (5)$$

The relation between object and image of a plane-parallel plate is

$$v=u-t/n. \quad (6)$$

By paraxial-ray tracing it can be shown that the corresponding relation for a lens is

$$v=\frac{u-t/n-d_1ut/n}{1+d_1u+d_2u-d_2t/n-d_1d_2ut/n}, \quad (7)$$

where

$$d_1=(n-1)/R_1, \quad d_2=(1-n)/R_2. \quad (8)$$

Let R_1 and R_2 become infinite in any manner. Then from (1), $1/f$ converges to zero and f becomes infinite.

The linear magnification converges to unity. For by use of (5) the magnification m can be written

$$m=s'/s=f/(f+s);$$
$$1/m=1+s/f=1+u/f+HV_1/f,$$

since

$$s=u+HV_1.$$

From (2)

$$\lim HV_1/f=\lim (n-1)t/nR_2=0.$$

Therefore

$$\lim 1/m=\lim m=1.$$

The image of a given axial object point always converges to the conjugate of the point with respect to the plate. For, from (8), d_1 and d_2 converge to zero, and (7) reduces to (6).

The expressions (2) and (3) can be written

$$V_1H=t/[n(1-R_2/R_1)-(n-1)t/R_1], \quad (9)$$

$$V_2H'=t/[n(R_1/R_2-1)-(n-1)t/R_2]. \quad (10)$$

It is necessary to consider three cases.

Case I

The ratio R_1/R_2 does not converge to any value, finite or infinite. Such a condition would exist, for example, if

$$R_2=R_1(2+\sin R_1).$$

In this situation the unit points do not approach definite locations.

Case II

The ratio R_1/R_2 converges to 1. Then the denominators in (9) and (10) approach zero, and

[1] K. Pestrecov, "Unit planes in an optical system including a plane-parallel plate," J. Opt. Soc. Am. **31**, 338 (1941).

[2] Arthur C. Hardy and Fred H. Perrin, *The Principles of Optics* (McGraw-Hill Company, 1932), pp. 56, 58–61.

V_1H and V_2H' become infinite. Inspection of (4) shows that the limit of HH' depends on the limit of R_1-R_2. Thus when $R_1=R_2$, one of the cases considered by Pestrecov, $\lim HH'=t$. Under other conditions HH' may approach any value, finite or infinite; or the limit may not even exist, as for example if $R_2=R_1+\sin R_1$.

Case III

The ratio R_1/R_2 converges to a value $\rho\neq 1$. Then

$$\lim V_1H=\rho t/n(\rho-1), \tag{11}$$

$$\lim V_2H'=t/n(\rho-1). \tag{12}$$

From (4)

$$HH'=t-\frac{(R_1/R_2-1)t}{n(R_1/R_2-1)-(n-1)t/R_2}$$

so

$$\lim HH'=t-t/n. \tag{13}$$

By proper choice of ρ, $\lim V_1H$ can be given any desired value; that is, the first unit point can be made to approach as limit any desired point on the optical axis. H' will then converge to the conjugate point with respect to the plate, as can be verified by substituting the limiting values from (11) and (12) for u and v, respectively, in (6).

The use of any such set of unit points in compounding the plate with another optical system gives the same results as those obtained by Pestrecov. Changing the notation slightly, let the subscript $(_1)$ denote points or quantities of the plate, and $(_2)$ those of the other system. Let H and H' be the unit points of the compound system, and let $H_1'H_2=a$. Then

$$\lim H'H_2'=\lim af_2/(f_1+f_2-a)=0$$

as f_1 becomes infinite. That is, the presence of the plate at one end of the system does not alter the position of the principal point at the other end. For the location of H, consider

$$HH_2=HH_1+H_1H_1'+H_1'H_2.$$

Since

$$HH_1=-af_1/(f_1+f_2-a),$$
$$\lim HH_1=\lim -a/[1+(f_2-a)/f_1]=-\lim a;$$

and as by definition $H_1'H_2=a$,

$$\lim HH_2=\lim H_1H_1'=t-t/n$$

from (13); so that the presence of the plate puts the first unit point a distance $t-t/n$ to the left of its position without the plate.

It is evident that so far as considering the plate as a limiting form of a lens is concerned, any pair of conjugate points of the plate may with equal justice be taken as the unit points of the plate.

Ninth Summer Conference on Spectroscopy and its Applications

THE ninth in a series of summer conferences on Applied Spectroscopy was held at the Massachusetts Institute of Technology on July 21 to 23, inclusive. 250 advance reservations, the limiting capacity of the hall, were received for the Conference. Most of the papers were followed by interesting and animated discussions. The program follows:

Monday, July 21, at 9.30 A.M.

Spectrographic Analysis of Materials

Residual Arc Spectra of 30 Chemical Elements Diluted in Silver. W. F. MEGGERS (with B. F. SCRIBNER), *National Bureau of Standards.*

Spectrograph Irradiation for Quantitative Analysis. M. SLAVIN, *United States Bureau of Mines.*

Application of Calculating Board Employing Gamma Curve for Quantitative Spectrographic Analysis. C. KING, *Harry W. Dietert Company.*

Some Spectrochemical Applications to Vacuum Tube Research. A. E. RUEHLE, *Bell Telephone Laboratories.*

Discussion of the Question of Plate Calibration Methods and Errors. R. A. SAWYER (with H. B. VINCENT and H. H. GROSSMAN), *University of Michigan.*

Monday, July 21, at 2 P.M.

Spectrographic Apparatus

Spectrochemical Analysis of Briquetted Test Specimens. H. W. DIETERT, *Harry W. Dietert Company.*

Use of Geiger-Müller Counters in Spectroscopy. O. S. DUFFENDACK (with W. E. MORRIS), *University of Michigan.*

New Large Two-Lens Quartz Spectrograph of Compact Design. S. JACOBSOHN, *Gaertner Scientific Corporation.*

New Microphotometer, H. B. VINCENT (with R. A. SAWYER), *University of Michigan.*

Infra-Red Spectrometer for Industrial Use. W. H. AVERY, *Shell Oil Company.*

Tuesday, July 22, at 9.30 A.M.

Metallurgical Analysis

Observations on the Spectrochemical Analysis of Tin. B. F. SCRIBNER, *National Bureau of Standards.*

Spectrographic Analysis of Four Percent Copper-Aluminum Alloys for Copper. H. V. CHURCHILL (with J. R. CHURCHILL), *Aluminum Company of America.*

Spectrographic Analysis of Iron for Tellurium. R. E. NUSBAUM (with J. W. HACKETT), *Research Laboratories Division, General Motors Corporation.*

Quantitative Determination of Calcium in the D.C. Arc. D. NORMAN (with W. W. A. JOHNSON), *New England Spectrochemical Laboratories.*

Spectrochemical Determination of Lead, Cadmium, and Zinc in Dusts, Fumes and Ores. H. I. OSHRY (with JAMES W. BALLARD AND H. H. SCHRENK), *United States Bureau of Mines.*

Tuesday, July 22, at 2 P.M.

Applications of the Spectrograph to Toxicological Investigations. C. W. RANKIN, *New York State Police.*

Time Effects and Arc Intensities. H. HEMMENDINGER, *Massachusetts Institute of Technology.*

Film and Plate Processing Equipment. J. A. SCHUCH, *Harry W. Dietert Company.*

Spectrographic Study of Meteorites. W. W. A. JOHNSON (with D. NORMAN), *New England Spectrochemical Laboratories.*

Preparation of Purified Inorganic Compounds for Use in Spectrographic Standards. R. C. HUGHES, *University of Florida.*

Tuesday Evening: Informal Dinner

Wednesday, July 23, at 9.30 A.M.

Absorption Spectroscopy

Automatic Recording of Absorption Spectra and Rotatory Dispersion. W. R. BRODE (with C. H. JONES), *Ohio State University.*

Blank and Background Effect on Photographed Spectral Lines. L. W. STROCK, *Simon Baruch Research Institute.*

Absorption Spectra of the Aromatic Amino Acids and of Certain Purified Proteins. H. H. DARBY, *Columbia University.*

Analysis of Toluene for Benzene by Ultraviolet Absorption of Vapor Mixtures and the Determination of Concentration of Toluene in Air by an Interferometric Method. P. A. COLE (with D. W. ARMSTRONG), *United States Public Health Service.*

Apparatus for the Study of Rapid Chemical Reactions. D. DUBOIS, *Yale University School of Medicine.*

New Quartz Spectrophotometer. A. O. BECKMAN (with H. H. CARY), *National Technical Laboratories.*

Wednesday, July 23, at 2 P.M.

General Spectroscopy and Discussion

Strongest Lines of Singly Ionized Atoms. W. F. MEGGERS, *National Bureau of Standards.*

On the Characteristic Function of the Photographic Plate, II. Application to Analysis. J. SHERMAN, *United States Navy Yard.*

Naming Spectroscopic Analytical Methods. M. G. MELLON, *Purdue University.*

Discussion on Units for the Reporting of Research on Absorption Spectra. H. H. DARBY, *Columbia University.*